Product Variety and Competitive Discounts*

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A market with free entry monopolistic competition is studied. Nonlinear pricing is shown to be the Bertrand-Nash equilibrium strategy for firms. Given small per capita fixed costs, the nonlinear pricing equilibrium approaches the perfectly competitive equilibrium with marginal cost pricing. Nonlinear pricing is associated with greater product variety than linear pricing. Increased variety leads to efficient pricing. *

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1. INTRODUCTION

Nonlinear pricing is frequently observed in markets with differentiated products. Quantity discounts or premia are given through entry fees, coupons, package sizes, or product lines. In this paper, a model of monopolistic competition with free entry of firms and nonlinear pricing is presented. It is shown that product variety and quantity-dependent pricing are closely linked in a number of interesting ways. This association has important consequences for the asymptotic efficiency of monopolistic competition.

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In a market with differentiated products it may be natural to assume that each consumer has a most preferred good in the space of product characteristics. For convenience, the commodities set and consumer preferences are given a spatial representation. In this setting, consumers whose most preferred good is a close substitute to the brand offered by a firm have a greater marginal willingness to pay than those consumers for whom the firm's brand is a distant substitute. Firms are unable to observe each consumer's most preferred brand but firms know the distribution of consumer preferences. As a result, the firm's profit maximizing pricing policy is to offer consumers a nonlinear price schedule. Consumers self-select by the size of their purchase and thereby reveal the characteristics of their most preferred brand.

In the presence of competition from rivals offering differentiated brands, it is shown that nonlinear pricing is also the firms' optimal strategy at a Bertrand-Nash equilibrium. The outlay schedules serve not only to separate consumers on the basis of their most preferred commodity, but also determine how competing brands will divide the market. Thus, with differentiated products nonlinear pricing appears as an equilibrium strategy.

At the same time, since nonlinear pricing increases profits as compared to linear pricing, it is shown that additional entry occurs, leading to a greater number of brands being offered. Thus, nonlinear pricing can be seen as a source of increased product variety.

Increased product variety allows consumers to obtain close substitutes for their most preferred good. This reduces the incentives for price discrimination for firms. In the limit, as fixed costs become small, the monopolistically competitive equilibrium is shown to approach the competitive equilibrium with marginal cost pricing.

The nonlinear pricing equilibrium is presented in Section 2 and its properties are examined in Section 3. Section 4 explores the connections between variety and efficiency. The equilibrium with free entry is characterized for an example in Section 5. Nonlinear and linear pricing are then compared in Section 6. Conclusions are given in Section 7.

2. A Model of Monopolistic Competition

2A. Consumers

The set of available brands is represented by locations \( l', j = 1, \ldots, m \) in a circular brand space of unit length. Consumers choose to purchase a

\footnote{For differentiated product models of this type see, for example, Salop [20] and Novshek [17].}
single brand \(j\). Each consumer has a most preferred good with characteristics \(l^*\). Consumers’ most preferred goods are uniformly distributed in the brand space with density \(D\). For necessary and sufficient conditions under which a product set and a set of preference orderings on that product set can be represented as a circular product market, see Horstman and Slivinski [11].³

A consumer’s utility depends on the number of units purchased, \(q^j\), and on the distance⁴ between the brand’s characteristics and the consumer’s most preferred brand, \(|l^* - l^i|\), \(U = U(q^j, |l^* - l^i|) + y\), where \(y\) is a numeraire commodity. The consumer’s most preferred good \(l^*\) is his private information. Firms therefore cannot observe the distance between their brand and the consumer’s most preferred brand although the distribution of consumer preferences is common knowledge.

Each firm \(j\) offers consumers a nonlinear price schedule \(P^j(\cdot)\), \(j = 1, \ldots, m\). Each consumer chooses a brand \(j\) and a purchase \(q^j\). The consumer’s net benefits from purchasing brand \(j\) are

\[
S^j(r) = \max_{q} \left[ U(q^j, r) - P^j(q^j) \right], \quad (1)
\]

where \(r = |l^* - l^i|\). Let \(q^j(r) = q^j(r, P^j(\cdot))\) denote the solution to (1). We will verify that the consumer’s problem (1) is well defined. Then, the consumer selects from the available brands to maximize net benefits, \(j^*(l^*) = \arg \max_j S^j(|l^* - l^i|)\).

Consumer’s preferences are assumed to be represented by \(U(q, r) = \int_0^r v(x, r) \, dx\). Marginal willingness to pay, \(v\), is twice continuously differentiable and decreasing in \(q\) and \(r\). Thus, the quality of the product is a substitute for the quantity consumed. Let \(U, \equiv \partial U(q, r)/\partial r, v_q \equiv \partial v(q, r)/\partial q, v_r \equiv \partial v(q, r)/\partial r\). Let demand for \(q\) be normal in \(r\), \(v_q r < 0\). Finally, without loss of generality, \(v\) may be parameterized so that \(v\) is concave in \(r\). The preceding assumptions guarantee that a complete separating equilibrium exists. This is for convenience only. Most of the results go through with little change if we allow pooling of consumers across types associated with different most preferred goods. Finally, we rule out consumer retrading of goods or arbitrage.

By standard application of the revelation principle,⁵ the output-payment

³ The necessary and sufficient conditions stated by Horstman and Slivinski [11] for the unit density are as follows. First, for every consumer there is exactly one other consumer with an opposite preference ordering. This rules out the single-peakedness condition that is often used in social choice. Second, let consumers \(a\) and \(d\) have opposite preferences and let \(c\) and \(e\) have opposite preferences. Then, if a consumer at \(b\) is indifferent between goods at \(a\) and \(c\) he is also indifferent between goods at \(e\) and \(d\).

⁴ Either Euclidean or arc distance may be used, see Horstman and Slivinski [11].

⁵ On the revelation principal, see Myerson [15] and Dasgupta, Hammond, and Maskin [4].
COMPETITIVE DISCOUNTS

FIGURE 1

schedules $q^j(r)$, $P_j(q^j(r))$ may be represented by a direct mechanism $q^j(r)$, $R^j(r)$ for which consumers truthfully announce the distance between brand $j$ and their most preferred brand and are assigned an output-payment pair $q^j, R^j$. Essentially, consumers self-select by revealing the characteristics of their most preferred brand. Then, by well-known arguments,\textsuperscript{6} we have the following necessary conditions. The proof is given in the Appendix.

**Lemma 1.** Given incentive compatibility, the output-schedule $q^j(r)$ and net benefit schedule $S^j(r)$ are nonincreasing in $r$. Also, $\frac{\partial S^j(r)}{\partial r} = U_j(q^j(r), r)$.

The characterization of net benefits in Lemma 1 is crucial to our analysis of monopolistic competition. Let brands $j$ be numbered clockwise in ascending order, $j = 1, 2, ..., m$ around the brand space. Consider the customers between brands $j$ and $j+1$. The price schedules $P_j$ and $P_{j+1}$ induce a partition\textsuperscript{7} of consumers with preferred brands in the interval $[l^j, l^j+1]$.

**Lemma 2.** The interval $[l^j, l^j+1]$ may be divided into those who purchase brand $j$, $[l^j, B^j_+]$, those who purchase neither brand, $(B^j_-, B^j_+)$, and those who purchase brand $j+1$, $(B^j_+, l^j+1)$, with $B^j_+ \leq B^j_{j+1}$.

To obtain the lemma, note first that since $S^j$ is nonincreasing, if $S^j(B^j_+) = 0$ and $S^j+1(|I^j+1-B^j_{j+1}|) = 0$ and $B^j_+ \leq B^j_{j+1}$ the lemma is satisfied. In this case brands are sufficiently differentiated that the firms are local monopolists, see Fig. 1a. Otherwise, if brands are close substitutes, firms compete with neighboring brands and their markets are divided by a consumer exactly indifferent between the two brands at $B^j_+ = B^j_{j+1}$, see Fig. 1b. Therefore $S^j(r) \equiv S^j(|I^j+1-r|)$ as $r \equiv B^j_+ = B^j_{j+1}$ for $r \in [l^j, l^j+1]$.

\textsuperscript{6} See, for example, Mirrlees [14], Guesnerie and Laffont [8], and Maskin and Riley [13].

\textsuperscript{7} Since consumers have measure zero, we could employ closed intervals.
and the lemma also holds in this case. In this case, competition requires firms to take into account the price schedules of rivals in selecting their price schedules.

2B. Equilibrium

It is required that the number of firms \( m \) be integer valued. Firm cost functions are given by \( C(Q) = F + kQ \), where \( F > 0 \) is a fixed cost of production and \( k > 0 \) is marginal cost. Each firm supplies a single brand. We allow free entry and exit of firms in the brand space.

Firm strategies consist of a brand location \( l' \) and a price schedule \( P^j(\cdot) \). A common problem in similar models of monopolistic competition with differentiated products is that firms may choose location and price such that a firm with a closely related product makes no sales. For example, a firm may choose the same product characteristics as another firm and undercut the price by epsilon. This can easily led to nonexistence of equilibrium as prices collapse to average cost which leads firms to raise prices again.

The present analysis considers a two-stage game in which the problem of undercutting does not appear in equilibrium. In the first stage, firms commit to brand locations \( l' \). In the second stage firms compete with price schedules \( P^j(\cdot) \). The perfect equilibrium of the two-stage game consists of a market structure \( j = 1, \ldots, m^* \) and a set of strategies \( (l^{j*}, P^{j*}) \) such that the following apply: (i) In the second stage game, given locations \( l^{j*}, j = 1, \ldots, m \), firm price schedules \( P^{j*} \) are chosen to maximize profits at a Bertrand-Nash equilibrium. (ii) In the first stage, all firms in the market must anticipate nonnegative profits. (iii) There is free entry in the first stage and any additional entrant \( (m^* + 1) \) earns negative profits.

3. Competitive Price Schedules

The second stage equilibrium first is characterized for a given market structure \( m^* \) and given firm locations \( l^{j*}, j = 1, \ldots, m^* \). Then, the first stage equilibrium strategies and the resulting second stage outcome are considered. There are two types of second stage equilibria which correspond to the cases of local monopoly and competition given in Lemma 2. Conditions are given under which each equilibrium is observed.

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8 One approach to this problem is the assumption of modified zero conjectural variation given in Novshek [17]. This allows a firm to take the strategies of all other rival firms as given unless that firm's price affects sales to consumers whose most preferred good is a rival's brand. The rival will respond to being undercut for sales to consumers at its own location in brand space. An interpretation of modified zero conjectural variation is that each firm responds to price cuts which take away customers who most prefer their brand. See Novshek [17] for additional discussion and references.
Consider first the local monopoly case. With sufficiently differentiated products, each firm chooses $P'(\cdot)$ subject to the individual rationality constraint $S'(r) \geq 0$ for all $r \leq B$, where $B$ is the firm's market boundary. It can be shown that the monopolist has incentive to raise the total outlays $P'$ until $S'(B) = 0$. Since $S'(\cdot)$ is nonincreasing the individual rationality constraints are satisfied. The firm's problem is then to choose its price schedule $P(\cdot)$ to maximize profits,

$$\max_{P(\cdot)} 2D \int_0^B \left[ P(q(r, P)) - kq(r, P) \right] dr. \quad (2)$$

This equilibrium is easily characterized.

**Proposition 1.** Local monopoly equilibrium.

(i) The optimal nonlinear price schedule for each firm is given by

$$P_M(q) = \int_0^q v(x, B^M) \, dx \quad \text{for } 0 \leq q \leq q(B^M),$$

$$P_M(q) = \int_0^{q(B^M)} v(x, B^M) \, dx + \int_0^q v(x, \rho(x)) \, dx \quad \text{for } q(B^M) \leq q,$$

where $\rho(x) = q^{-1}(x)$ for $x \geq q(B^M)$.

(ii) The demand schedule $q(r)$ is given by $q(r) = 0$ for $r > B^M$ and $q(r)$ solves

$$v(q, r) - k = -v_r(q, r)r,$$

for $0 \leq r \leq B^M$, $q(B^M) \geq 0$.

(iii) The unconstrained optimal market boundary $\bar{B}^M$ solves

$$\int_0^{q(B^M)} v(x, \bar{B}^M) \, dx - kq(\bar{B}^M) = -\bar{B}^M \int_0^{q(B^M)} v_r(x, \bar{B}^M) \, dx.$$  \quad (5)

Then, $B^M = \min(\frac{1}{2}, \bar{B}^M)$.

The main point made by Proposition 1 is that monopolistic competition with differentiated products entails price discrimination. In the absence of consumer arbitrage opportunities, we will observe quantity discounts rather than linear pricing. Linear pricing is generally taken as the optimal strategy for firms in these types of models. The proof of Proposition 1 is similar to the monopoly solution presented in monopoly nonlinear pricing models such as Spulber [23, 24], see also Maskin and Riley [13], and is not restated here.
Part (i) of the proposition states that the nonlinear outlay schedule follows the marginal consumer's marginal willingness to pay schedule for \( 0 < q < q(B^M) \) and so captures net consumer surplus at the market boundary. Part (ii) gives the consumer purchases as a function of the distance of the consumer's most preferred good from the brand purchased. Observe that for a consumer whose most preferred brand exactly coincides with the firm's brand the consumer pays exactly marginal cost on the last unit purchased. Let \( p(q) = dP(q)/dq \). Then,

\[
p(q(0)) = v(q(0), 0) = k. \tag{6}
\]

Finally, observe that part (iii) gives a rule for the monopoly market boundary. The rule requires that the net revenues obtained from the marginal consumer exactly equal the additional revenues from all inframarginal consumers which could be obtained by raising the outlay schedule and thus reducing the size of the market, that is, 

\[
P(q(B^M)) - kq(B^M) = \int_0^{B^M} \frac{dP(q(r))}{dB^M} dr.
\]

Note that a monopoly equilibrium in the second stage need not involve symmetric locations since monopoly market boundaries need not touch. It is now shown that symmetric locations will be observed with a competitive equilibrium in the second stage.

In a competitive equilibrium, firm \( i \)'s market boundaries \( B^i_+ \) and \( B^i_- \) will depend on its location \( l^i \) and on the equilibrium nonlinear price strategies of rivals, \( P^{i+1} \) and \( P^{i-1} \). The marginal consumers \( B^i_+ \) and \( B^i_- \), as in Fig. 1b, are defined by

\[
S'(B^i_+) = S'^I(|l^{i+1} - B^i_+|),
\]

\[
S'(B^i_-) = S'^I(|l^{i-1} - B^i_-|), \tag{7}
\]

taking \( B^i_+ \) and \( B^i_- \) as the distance from \( l^i \). Clearly, specifying the total outlay schedule \( P(\cdot) \) implicitly determines the market boundaries. We may then find the optimal schedule for firm \( i \) using a two-step procedure.

From the consumer's problem, 

\[
P'(q^i(r)) = U(q^i(r), r) - S'(r),
\]

and similarly for \( B^i_- \). So, we have

\[
P'(q^i(r)) = U(q(r), r) - S'(B^i_+) + \int_{r'}^{B^i_+} U_{,r}(q^i(r'), r') dr'. \tag{9}
\]
We may also write $P' i$ in terms of $B'_i$. The firm’s problem is then to solve a maximization problem similar to Eq. (2) subject to $q'(r)$ nonincreasing in $r$ and the constraints in Eq. (7). Applying integration by parts to Eq. (9), and using Eq. (7), the competitive equilibrium strategy $P'^* i$ is obtained by choosing $q'$ as follows,

$$
\max_{q'} D \int_0^{B'_i} [U(q'(r), r) + U_r(q'(r), r)r - k(q(r))] dr
$$

$$
- DB'_i S'i^{i+1}(|l'i^{i+1} - B'_i|)
$$

$$
+ D \int_0^{B'_i} [U(q'(r), r) + U_r(q'(r), r)r - k(q(r))] dr
$$

$$
- DB'_i S'i^{i-1}(|l'i^{i-1} - B'_i|),
$$

subject to $q'(r)$ nonincreasing.

Consider the pointwise maximum in Eq. (10). It follows immediately that for $q' > 0$,

$$
v(q'(r), r) + v_r(q'(r), r)r - k = 0
$$

for $q'(r), r \in [0, B'_i]$ and $r \in [0, B'_+]$. Thus, the monopolist’s marginal price schedule solves $p(q) = v(q, \rho(q))$, where $\rho(q) = q^{-1}(r)$ for $q' > 0$. This has the important implication that competition does not affect the marginal price schedule, and hence consumer purchases, for consumers within the firm’s market boundary. The marginal price schedule for $q > \min\{q'(B'_i), q'(B'_+)\}$ is thus identical to the monopoly marginal price schedule (in Eq. (4)).

The preceding discussion implies that competition in price schedules is in terms of the height of the total outlay schedules. This is equivalent to competition in terms of market boundaries, $B'_+, B'_-$. Thus, profit, as given in Eq. (10), is entirely dependent on the locations and market boundary choices. This implies that in the first period entry equilibrium firms will attempt to secure the largest market by locating between firms which are spaced the farthest apart. This implies symmetric locations in the first stage.

Therefore, it has been shown that the competitive entry model is related to a model in which consumers buy either one unit or none of a preferred brand and consumer utility functions are monotonically decreasing in the distance between the consumer’s location and that of his chosen brand. In this setting, the unit prices set by firms determine market boundaries. I owe this observation to an anonymous referee. It should also be emphasized that nonlinear pricing involves different total payments for each consumer. Consumers face different marginal prices depending on the size of their purchases. Thus, consumers can be viewed as facing a different per unit price $p(q(r))$ and a different “fixed fee” $[P(q(r)) - p(q(r)) q(r)]$ depending upon the distance between the purchased brand from their most preferred brand.
equilibrium if the second stage is competitive. Symmetric locations then allows a simple characterization of second period competition. It can be shown that in equilibrium market boundaries are symmetric and maximize profits as in Eq. (10),

\[ U(q'(B^*), B^*) + U_*(q'(B^*), B^*) B^* - kq'(B^*) - S^{i+1}(|I^i + 1 - B^*|) + B^* S^{i+1}(|I^i + 1 - B^*|) = 0. \]  

(11)

From the consumer's problem, and \( q^i = q^j \),

\[ S^{i+1}(|I^i + 1 - B^*|) = U_*(q(|I^i + 1 - B^*|), |I^i + 1 - B^*|). \]  

(12)

By symmetry of locations it follows that \( I^i + 1 - B^* = B^* = 1/2m \) so that \( S^{i}(B^*) = U_*(q(B^*), B^*) \) for all \( i \). Thus, the outlay schedule is fully determined. From Eqs. (7), (9), (11), and (12), and \( P(q) = P(q(B^*)) + \int_{r}^{p} P(q(r')) dr' \), we obtain

\[ P^*(q) = \int_{0}^{q(B^*)} [k - 2v_*(x, B^*) B^*] dx + \int_{q(B^*)}^{q} v(x, \rho(x)) dx. \]  

(13)

The preceding discussion is summarized by the next proposition. It establishes that nonlinear pricing arises as a result of optimal strategies at a monopolistically competitive equilibrium.

**Proposition 2.** Competitive equilibrium.

(i) The optimal nonlinear price schedule for each firm producing at the competitive equilibrium is

\[ P^*(q) = \int_{0}^{q} (k - 2B^* v_*(x, B^*)) dx \quad \text{for } 0 \leq q \leq q(B^*), \]

\[ P^*(q) = \int_{0}^{q(B^*)} (k - 2B^* v_*(x, B^*)) dx + \int_{q(B^*)}^{q} v(x, \rho(x)) dx \quad \text{for } q(B^*) \leq q, \]  

(14)

for \( B^* = 1/2m \).

(ii) The demand schedule is given by \( q(r) = 0 \) for \( r > B^* \) and \( q(r) > 0 \) solves

\[ v(q, r) - k = -v_*(q, r) r, \]  

for \( 0 \leq r \leq B^*, \) \( q(B^*) \geq 0 \).

Proposition 2(ii) is proved in the Appendix.
From Propositions 1 and 2 observe that under either local monopoly or competition, self-selection by the quantity purchased implies that those consumers whose most preferred good is a close substitute for the brand purchased pay more than those for which the purchased brand is less desirable.

Given the characterization of local monopoly and competition, necessary and sufficient conditions can be given under which the two types of equilibria will be observed with free entry. Recall that the number of firms is an integer. The result is comparable with Novshek's [17] result for monopolistic competition with linear pricing in a linear demand model. If $m^*2B^M < 1$, then all firms will be local monopolists. In this case, the spacing of firms will not affect the equilibrium as long as market boundaries do not overlap. Thus, as already noted, the array of brands is not unique for this case. Otherwise, the monopolistically competitive equilibrium will be unique.

**PROPOSITION 3.** At the competitive equilibrium with nonlinear price schedules and market structure an integer $m^*$, the following hold:

(i) Firms operate as local monopolists if and only if $m^* \leq \max\{1, 1/2\tilde{B}^M\}$.

(ii) If $m^* = 1 > 1/2\tilde{B}^M$, the monopolist operates with a constrained market boundary equal to 1/2.

(iii) If $1 < m^* < 1/2\tilde{B}^M$, all firms are local monopolists with unconstrained market boundaries $B^M$.

(iv) If $m^* > \max\{1, 1/2\tilde{B}^M\}$ there is a unique symmetric competitive equilibrium with market boundaries $B^* = 1/2m^*$.

It is important to observe that all consumers are served under competition. Competition serves to constrain the markets of firms and is associated with greater product variety. The purchase of the marginal consumer is raised since $q(r)$ is nonincreasing, $q(B^*) > q(B^M)$. Thus, we have as an immediate consequence of Proposition 3,

**COROLLARY 1.** Total market output is greater under competition than local monopoly.

4. **VARIETY AND EFFICIENCY**

The effects of increased variety on the equilibrium are now studied. Increasing variety allows consumers to purchase goods whose characteristics closely resemble their most preferred good. Because each brand's
market radius is reduced, firms face an increasingly homogeneous market. Consumer preferences are within a narrow range and the effects of product characteristics on consumer marginal willingness to pay within the firm's market are reduced.

We now give a sufficient condition on demand for quantity discounts to occur. Quantity discounts are said to exist if the marginal price schedule is downward sloping.

**Lemma 3.** If \( v_q(r)/r \) is nondecreasing in \( r \), then quantity discounts are offered by firms, \( \partial p(q)/\partial q < 0 \).

**Proof.** From Eq. (15), \( \partial p(q)/\partial q = q_v + v_r/(\partial q/\partial r) \). Thus, \( \partial p(q)/\partial q = [v_r(v_q - v_{rq}r) + v_{rr}v_q r]/[2v_r + v_{rr}] \). The denominator is negative since \( v_r < 0, v_{rr} < 0 \). By hypothesis, \( v_q - v_{rq}r \) is nonpositive. So, the numerator is positive.

Therefore, as the consumer's most preferred brand approaches that offered by a firm, the consumer purchases more and pays a lower marginal price. With greater variety, each consumer is able to purchase a brand closer to his most preferred good. This yields the following.

**Proposition 4.** Given quantity discounts, increased variety is associated with greater total output and lower output per firm.

Although output per firm falls since each firm serves a smaller number of consumers, the output per consumer increases yielding greater total output. See the Appendix for a proof.

The issue that is now addressed is whether with sufficient variety, the monopolistically competitive equilibrium with nonlinear pricing approximates perfect competition. It is shown that this is indeed so. This analysis provides the basis for a later result which shows that with sufficiently small fixed costs monopolistic competition is approximately competitive. As noted before, the consumer whose most preferred good coincides with the firm's brand pays just marginal cost \( k \) on the last unit purchased. As variety increases, the marginal price paid approaches \( k \) for all consumers. Further, the fixed fees paid by all consumers approach zero. The following result is for general forms of the outlay schedule and does not depend on whether quantity discounts are offered.

**Proposition 5.** As variety increases, the equilibrium with nonlinear pricing approaches the competitive equilibrium with price equal to marginal cost.

From Eq. (14), as \( m \to \infty \), \( P^*(q) \) approaches \( kq(0) + \int_{q(0)}^{q} v(x, \rho(x)) \, dx \) for \( q \geq q(0) \). But, since \( q(0) \geq q(r) \geq q(B^*) \), as \( B^* \) goes to zero, all con-
sumer purchases approach $q(r)$. So, consumers pay only marginal cost $k$. This implies that as products have sufficiently close substitutes, the Nash equilibrium nonlinear price schedules have reduced quantity discounts and approach a constant per unit marginal cost price.

It is interesting to contrast the preceding result with an alternative concept of competition given by the Löschian competitive equilibrium. In the Löschian free entry model, monopolists enter the market without recognition of competition from firms supplying close substitutes. Thus, firms behave as monopolists with smaller markets.

**PROPOSITION 6.** As variety increases in the Löschian competitive model, the outlay of each consumer approaches his total surplus, and the marginal price paid approaches marginal production cost.

The result is easily obtained from Eq. (3) in Proposition 1, where $P^M(q(0)) = \int_0^{q(0)} v(x, 1/2m) \, dx$. In the limit $P^M(q(0)) = \int_0^{q(0)} v(x, 0) \, dx$, the total surplus of the consumer whose most preferred good is supplied by the firm. Thus, the limit of Löschian competition is first-degree price discrimination. The monopolist captures the entire surplus of the consumer at the market boundary.

5. EQUILIBRIUM WITH FREE ENTRY: AN EXAMPLE

Consider an example with a quadratic utility function. $U(q, l^* - l') = aq - \frac{1}{2}bq^2 - cq |l^* - l'|$. This function satisfies all of our assumptions. The demand schedule is then $q(r) = (c - k - 2cr)/\beta$. It is easy to see from Eq. (5) that the unconstrained monopoly market is given by $\bar{B}^M = (c - k)/2c$ and consumer demand is zero at the boundary. Thus, firms are local monopolists if and only if $m^* \leq \max \{1, c/(a - k)\}$.

A frequently observed property of monopolistic competition is that as fixed costs become small, the equilibrium approaches the competitive outcome, see especially Novshek [17]. We verify that this result holds for the example given above.

**PROPOSITION 7.** Given quadratic utility, as fixed costs per capita $F/D$ go to zero, the competitive equilibrium with nonlinear price schedules approaches the perfectly competitive outcome.

**Proof.** Given $m$ firms and a competitive equilibrium, per firm profits are given by

\[ \pi(m) - F = 2D \int_0^{1/2m} \left[ \int_0^{q(1/2m)} (k - (1/m) v(x, 1/2m)) \, dx \right. \\
+ \left. \int_{q(1/2m)}^{q(r)} v(x, \rho(x)) \, dx - kq(r) \right] \, dr - F, \]

for sufficiently large \( m, m > (5/4)(c/(\alpha - k)) \), \( \pi'(m) < 0 \). For \( F \) sufficiently small, there exists \( m(F/D) \) such that \( \pi(m(F/D)) - F > 0 \) and \( \pi(m(F/D) + 1) - F < 0 \), and \( m(F/D) \) nonincreasing in \( F/D \). Thus, \( m(F/D) \to \infty \) as \( F/D \to 0 \). The result follows by Proposition 5.

It is interesting to compare this result with similar observations in linear pricing models, such as Novshek [17], see also Hart [9] and Gabszewicz and Thisse [6]. Competitive entry eliminates both the slope of the marginal price schedules and any additional fixed fees as well.

6. NONLINEAR PRICING vs. LINEAR PRICING

Nonlinear pricing yields greater profits than linear pricing for a monopoly. It is not apparent whether firms will make greater profits from nonlinear pricing at a competitive equilibrium. The following shows that in the quadratic case, nonlinear pricing increases profits even under competition.

Let marginal cost \( k \) equal zero. Profit at the competitive equilibrium with market structure \( m^* \) equals

\[ \pi(m^*) = (Dc/\beta(m^*)^2)\left[ \alpha - (5/6)(c/m^*) \right] - F. \]

Profit at a linear pricing equilibrium with modified conjectural variation as calculated in Novshek [17] is

\[ \tilde{\pi}(m) = (Dc/\beta m^2)\left[ \alpha/8 - (25/16)(c/m) + (7/8)\left[ \alpha^2 - \alpha c/m + 13c^2/m^2 \right]^{1/2} \right]. \]

It can be shown that \( \pi(m) \geq \tilde{\pi}(m) \) for \( m \) sufficiently large \( (m \geq 5) \). This implies that for small fixed costs, nonlinear pricing yields greater profits than linear pricing at a free entry competitive equilibrium. This implies that there will always be greater incentives for additional entry if firms employ nonlinear pricing.
Proposition 8. For quadratic utility and small fixed costs, product variety is greater at a monopolistically competitive equilibrium with nonlinear price schedules than at a monopolistically competitive equilibrium with linear prices.

In linear-pricing market models, such as Salop [20], variety is often observed to be greater than the socially optimal level. If this is the case, then by Proposition 7, nonlinear pricing leads to additional excessive proliferation of brands. A similar conclusion is reached by Katz [12] with a continuum of firms and two consumer types. As Salop [20] emphasizes, however, the excess product variety observed may not be robust to changes in consumer demand assumptions.11

7. Conclusion

In the model of monopolistic competition we have studied, nonlinear pricing emerged as the equilibrium strategy of firms at a Bertrand–Nash equilibrium. The presence of competition did not alter the firms' marginal price schedules for those outputs at which sales are made, but determined the level of total outlays for each consumer. Necessary and sufficient conditions were obtained under which the free entry equilibrium is characterized by either local monopoly or by competition for consumers between brands. As the firm's fixed costs become small, or equivalently as the number of consumers becomes large, the free entry equilibrium was shown to approach perfect competition with marginal cost pricing. By decreasing product differences, entry reduces opportunities for separation of consumers based on substitutability between the firm's brand and the consumer's most preferred good. The analysis showed that nonlinear pricing is likely to occur in markets with differentiated products and that quantity-dependent pricing in turn leads to increased variety. While there may be social welfare losses from excessive brand proliferation, these should be balanced against welfare gains from quantity discounts.

Appendix

Proof of Lemma 1. Let \( S_i'(r, \hat{r}) = U(q_i'(\hat{r}), r) - R(\hat{r}) \). Then, incentive compatibility requires \( S_i'(r, r) \geq S_i'(r, \hat{r}) \) for all \( r \) in the feasible range. Let \( S_i'(r) = S_i'(r, r) \). Then \( U(q_i'(r), r) - U(q_i'(r'), r') \geq S_i'(r, r) - S_i'(r', r') \geq U(q_i'(r'), r) - U(q_i'(r'), r') \). Dividing by \( (r - r') \) and taking limits as \( r' \to r \), \( \partial S_i'(r)/\partial r = U'_i(q_i'(r), r) \). The inequality also implies \( \int_{q_i'(r)}^{q_i'(r')} v(x, r') \, dx \geq \int_{q_i'(r)}^{q_i'(r')} v(x, r') \, dx \). Since \( v \) is decreasing in \( r \), \( q_i'(r) \geq q_i'(r') \) for \( r \leq r' \).

11 See, in particular, Dixit and Stiglitz [5] and Spence [21].
Proof of Proposition 2(ii). In Eq. (10) consider the bracketed term, \( J(q, r) = U(q, r) + U_r(q, r)r - kq \). First we establish that \( J \) is concave in \( q \). Since \( J_{qq} = u_{qq}(q, r) + u_{q}(q, r)r \), \( u_{qq} < 0 \) and \( u_q < 0 \), \( J_{qq} < 0 \). Let \( \bar{q} \) be the unconstrained (pointwise) maximum of the objective function in Eq. (10). Then, either \( \bar{q} = 0 \) or \( \bar{q} > 0 \) and solves \( J_q = v(q, r) + v_r(q, r)r - k = 0 \). For \( \bar{q} > 0 \), \( \partial \bar{q}/\partial r = -[2v_r + v_{rr}] / [v_q + v_{qr}] \). Since \( J \) is concave, the denominator is negative. Since \( v_r < 0 \) and \( v_{rr} < 0 \) by assumption, \( \partial \bar{q}/\partial r < 0 \). Thus, \( \bar{q} \) is either zero or if positive, it is strictly decreasing in \( r \). Further, \( \bar{q} \) is either positive at zero or zero everywhere. Thus, the boundary condition that \( q(r) \) be nonincreasing is satisfied and the unconstrained maximum solves (10).

Proof of Proposition 4. Output per firm with market structure \( m \) is \( Q_j(m) = 2m \int_0^{1/m} q(r) \, dr \) which is decreasing in \( m \). Total output is then \( Q(m) = m2 \int_0^{1/m} q(r) \, dr \) which is increasing in \( m \) since \( q(r) \) is decreasing in \( r \).

References

9. O. HART, Monopolistic competition in a large economy with differentiated products, Rev. Econ. Stud. 46 (1979), 1-30.


