Pricing of Cognitive Radio Rights to Maintain the Risk-Reward of Primary User Spectrum Investment

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Abstract—Cognitive Radio (CR) has been recently proposed as a method of alleviating the shortage of radio spectrum, by increasing the efficiency of spectrum use. However, as large portions of spectrum remain under long-term licenses, the economic welfare of the primary license holders must be taken into account, when considering methods of spectrum access that may degrade license holder Quality-of-Service (QoS) and therefore revenue. Several price discovery methods have been proposed to find the fee that license holders should charge for cognitive access to their spectrum. This paper examines the spectrum licenses themselves as an investment class. By performing a reward-to-variability (Sharpe Ratio) analysis of the spectrum license under different levels of CR activity, a floor price for CR access is derived such that the quality of the license holder’s spectrum investment from a Sharpe Ratio point of view is not degraded. An example scenario is provided to illustrate this pricing mechanism, and simulation results illustrate its effectiveness in maintaining the quality of the license holder’s spectrum investment.

Keywords—Cognitive radio; spectrum pricing; return-on-investment; Sharpe ratio

I. INTRODUCTION

Radio spectrum is a scarce resource, and traditional methods of fixed spectrum allocation through long term licenses have resulted in substantial inefficiencies in the use of some spectrum bands. A number of solutions have been proposed to alleviate the spectrum shortage by improving the efficiency of spectrum use, the most promising being Cognitive Radio [1], Radio Spectrum Markets [2] - [4] or a combination of both [5] – [7]. Cognitive Radio (CR) is an intelligent wireless communication technology that has the potential to increase the efficiency of spectrum utilization by cognitively exploiting underserved spectrum [1]. This is achieved by Secondary Users (SUs) accessing spectrum licensed to Primary Users (PU) under an opportunistic or a spectrum sharing scenario. The introduction of a radio spectrum market [2] - [4], or shorter term licenses [8], [9] for some spectrum bands aims to let market forces ensure that spectrum is efficiently allocated to the users that require it (and are willing to pay) the most.

Numerous communication providers have recently purchased licenses covering large portions of usable spectrum at significant cost. For example, in April 2000, 5 major UK mobile providers purchased licenses covering the third generation (3G) mobile spectrum expiring in December 2021 for £4 - 6 billion [10]. The consequences of the substantial remaining time and the massive up-front cost of such licenses are that without adversely affecting license holders, full deregulation of radio spectrum access cannot occur for some time. Also any attempt at altering the methods that licensed spectrum is accessed will have to consider the economic impact on the license holders and ensure that the potential returns-on-investments (ROI) made in spectrum licenses are not degraded.

A. Cognitive Radio

Cognitive Radio enables SUs to exploit spectrum bands that are currently un-occupied by PUs or for SU communication to co-exist with PU traffic under an interference constraint [1]. Consequently, additional utility can be gained from underutilized licensed spectrum. Substantial research has been undertaken into the technological issues associated with CR, including spectrum sensing [11], power control [12], [13], MAC design [14], and application to various network environments. A key advantage of Cognitive Radio is that there is no requirement for major alteration of PU service provision and operation.

However, most proposed CR implementations will result in PU QoS being affected to some degree by CR enabled SUs operating in the PUs’ spectrum bands. For example, the CR power allocation method proposed in [13], operates under a PU outage constraint; implying that PU QoS will be affected. Under an opportunistic access scheme, causes of PU QoS degradation include sensing error (eg. SUs incorrectly sensing PU occupied channels as free) or a lack of synchronization between PU and SU traffic (eg. PU begins transmitting immediately after the conclusion of an SU sensing period). The spectrum sharing access model will result in PU QoS degradation by reducing the signal-to-noise ratio of PU transmissions. PU QoS degradation can be manifest by reduced bandwidth available for the PU, and could lead to effects such as call dropping (for mobile providers), ultimately reducing revenue for PUs, and lowering the ROI for license holders. Without some form of compensation, there is no incentive for spectrum license holders to permit CR enabled SUs to operate in their spectrum bands.

B. Spectrum Markets

Radio Spectrum Markets offer methods of dynamic and efficient spectrum allocation and possible compensation for existing spectrum license holders, however, most proposals...
would require a substantial alteration of the method that PUs provide services and earn revenue. In [2], a price based spectrum allocation scheme is proposed, where the PUs offer spectrum to end users and end users can obtain service from any PU. The spectrum price and allocated bandwidth is determined by end user utility. If completely applied to existing licensed spectrum bands, such an approach would not take into account the need for PUs to obtain an ROI on their existing spectrum licenses. However, such a framework could be applicable to PUs trading currently unused (and therefore not revenue generating) spectrum.

In [3], the authors present an analysis of price dynamics in a competitive spectrum market, where spectrum access is traded in short time blocks. The market model in [3] accounts for different classes of spectrum providers and different end user requirements and could be feasible with existing long term licenses. License holders would become spectrum sellers and therefore could set prices such that their profit is maximized in a competitive environment, enabling license ROI to be maintained. Trading of spectrum between PUs is proposed in [4], where derivative contracts are used to alleviate short-term spectrum supply and demand imbalances among PUs. However, a derivative market would require an active spectrum spot market to enable price discovery. Unlike Cognitive Radio, the Spectrum Market approaches would require a substantial shift from the current vertically integrated model of wireless service provision, if it is to be implemented across the licensed spectrum bands. Even if the existing long term licenses are preserved, the role of PU spectrum license holders would shift to that of spectrum vendors (analogous to owners of electricity transmission networks). Under this new mode of operation, PUs may require a significantly altered business models to ensure an adequate spectrum license ROI is maintained.

A broker based spectrum management scheme is proposed in [8]. The proposed spectrum broker provides short term leases to spectrum users in a particular area based on centralized demand aggregation under various spectrum pricing mechanisms and network infrastructure scenarios. However such a spectrum management method conflicts with the long term licenses held by PUs, unless the spectrum brokers are the license holders themselves. The FCC’s Secondary Market Initiative [9] also aims to facilitate spectrum leasing arrangements and the transfer of spectrum licenses for shorter periods of time. Such methods may increase medium term efficiency of spectrum use, without the need for a complete shift in the wireless service model.

C. Cognitive Radio Combined with Market Strategies

As the Cognitive Radio approach requires far less modification of the PU spectrum license holders’ functionality, it seems like a more realizable method to alleviate spectrum shortages in the more immediate future (at least until the existing long term spectrum licenses expire). The potential standardization of CR techniques in the upcoming IEEE 802.22 protocol [15], suggests that deployment of CR enabled devices may occur soon. However, there still remains unresolved the issue of an adequate framework for compensation of PUs for the degradation in QoS that will result from CR access. Several approaches have been proposed by Niyato and Hossain in [5], [6] and [7] that combine SUs operating in PU channels under a CR model with market based methods for obtaining the level of compensation that PUs should receive.

In [5], the authors present a dynamic game-theoretic approach to determining the optimal price for Cognitive Radio access to PU licensed spectrum under a price competitive oligopoly market. Using a Bertrand game model, the authors analyze the spectrum price, PU profit and SU utility. A PU cost function is also incorporated to reflect the lost revenue due to QoS degradation resulting from CR-SU access. This analysis is extended in [6], where 3 different pricing models are considered: market-equilibrium, competitive and cooperative. A solution for the price under each model is presented along with possible distributed implementations and an analysis of the price behavior under various network scenarios. The results in [6] show the region under which the PUs will achieve a greater profit from permitting CR enabled SUs to operate than without any spectrum sharing. In [7], the market-equilibrium pricing model is further analyzed in the context of CR technologies and possible spectrum market structures.

While [5] - [7] present a solid framework for the discovery of an optimal price that PUs should charge for permitting SUs to operate in their spectrum, all of the analyses assumed that the PU cost function is deterministic. The formulation of the PU cost function in [5] - [7] implies that the effect of permitting CR users to operate in PU spectrum results in a decrease of bandwidth available for the provision of revenue generating services by PUs. The cost of permitting CR users to operate is defined as a function of reduction in revenue resulting from the PUs’ per-customer bandwidth requirements not being met. However, from results presented in [11]-[14], under both opportunistic access and spectrum sharing schemes, it is evident that the effect of CR SU operation on the reduction of bandwidth available for PU services (therefore reduction of PU revenue) can be a random process. For example, this will occur if SUs commit sensing error with some probability \( p_{err} \), or if the PU transmission slots and SU sensing periods are not synchronized. Randomness may also be evident in the bandwidth required by the PU to provide services, due to the stochastic processes that characterize demand for wireless services (eg. call arrival and duration at a mobile base station). Such randomness introduces a risk (variance) that the levels of lost PU revenue and will be different from the predicted values, and this should be taken into account if the economic welfare of PUs is to be properly accounted for.

In this paper, we present a method of valuation of CR rights by viewing the PU spectrum license as an investment. The expected return on the spectrum license investment depends on the expected demand for PU services (eg. mobile calls), while the stochastic nature of the demand results in volatility (variance) of returns. Since the variance of returns has been shown to be just as significant as mean returns to investor welfare [16], we analyze the change in the reward-to-variability ratio (Sharpe Ratio [17], [18]) of the spectrum license investment due to the operation of CR SUs. Consequently, we find a floor on the price that a PU should charge the SUs for rights to cognitively access its spectrum in the case of a PU license holder monopoly, or a floor on the spectrum price that determines if a PU should participate in a CR competitive
spectrum market (such as in [5] – [7]). Below this price, the reward-variability characteristics of the spectrum license investment fall below a level that a rational investor would accept, hence the PU should not offer its spectrum to CR users for any less.

We also illustrate the functionality of our valuation method in the simple scenario of a PU being a mobile base station, earning revenue from ongoing calls. We show how the presence of CR SUs, which introduces a probability of PU active calls being dropped early (due to collision with SU traffic), reduce base station revenue and change the variance of the revenue. We also show that the PU will not necessarily need to be fully compensated for lost revenue resulting from CR SU activity.

The rest of the paper is organized as follows. Section 2 explains the Rational Investor Model that we will assume that PUs adhere to. In section 3, we derive the coupon that should be paid by a CR SU, as a function of the change in expected return and variance of return that is experienced by the PU due to SU activity. The example scenario that we use to illustrate the function of our valuation method is defined in section 4, while in section 5 we analyze the effect of SU CR activity on PU revenue in the example scenario and the price that should be paid for CR access. Section 6 concludes the paper.

II. REWARD-TO-VARIABILITY AND THE RATIONAL INVESTOR MODEL

Markowitz Portfolio Theory [19] characterizes assets and portfolios of assets according to their expected return and variance of return. The reward-to-variability or Sharpe Ratio [17], [18] encapsulates those two measures of asset performance with respect to a reference asset (normally a risk-free asset such as short-dated government debt, earning the risk-free rate). In financial and economic theory, it is generally accepted that rational investors are risk-averse [19], [20]. When faced with 2 possible investment strategies where volatility of returns is a good measure of risk, a rational, risk-averse investor would tend to prefer the one with the higher Sharpe Ratio [20]. The Sharpe Ratio of an asset or portfolio with return \( r \) is:

\[
S = \frac{E(r) - r_f}{\sigma(r)}
\]

where \( r_f \) is the risk-free rate.

We assume that the PU (eg. a telecommunications company) is a risk-averse investor, and that its investment in communications infrastructure and spectrum is the best possible investment it could make from a rational investor and Sharpe Ratio point of view – otherwise it would not make the investment. To simplify the analysis, we assume that the PUs investments in communications infrastructure consist only of spectrum licenses and associated wireless infrastructure (referred henceforth as the spectrum asset or \( \Phi_{spectrum} \)). We also assume that the PU’s holding of other investment types (eg: shares, bonds, etc…) are small compared to its investment in assets it uses to provide wireless services. However, the PU may have significant cash holdings, assumed to be earning the risk free rate.

The PU’s return on the spectrum asset \( r_{spectrum} \) is characterized by the revenue it earns from customers using its wireless services, such as mobile voice calls or data access. The volatility or returns is due to the stochastic nature of wireless service demands (eg. random call arrivals, random call duration). We assume that per-period billing is available for some small period \( \Delta t \) and that the revenue \( R \), earned per period of each ongoing call (or data session) incorporates all costs associated with providing one period of connectivity (any call initiation/flag-fall charges are assumed to be included in \( R \)). Therefore, a PU’s per-period return on the spectrum asset is:

\[
r_{spectrum}(t) = \frac{n(t)R}{K}
\]

where \( n(t) \) is the number of ongoing calls during period \( t \), and \( K \) is the price of the spectrum asset (spectrum license fee, infrastructure cost). Assuming that the risk free rate \( r_f \) is divisible into \( \Delta t \) period intervals\(^1\), the Sharpe Ratio of the per-period spectrum asset return is:

\[
S_{spectrum} = \frac{E(r_{spectrum}) - r_f}{\sigma(r_{spectrum})}
\]

where:

\[
E(r_{spectrum}) = \frac{R}{K} E(n) \text{ and } \sigma^2(r_{spectrum}) = \left( \frac{R}{K} \right)^2 \sigma^2(n)
\]

The results of [21] can be used to obtain the Sharpe Ratio of multi-period returns (eg. daily, annual). If the numbers of active calls during each period are IID, the \( q \)-period Sharpe Ratio is [21]:

\[
S_{spectrum}(q) = \sqrt{q} S_{spectrum}
\]

From (3), it is evident that any factor that affects the mean \( E(n) \) and variance \( \sigma^2(n) \) of active connections would also affect the Sharpe ratio of the spectrum asset \( \Phi_{spectrum} \) and therefore its desirability as an investment. By utilizing the Sharpe Ratio as the measure of investment quality, we can encapsulate the effects of CR SU activity on the mean and variance of returns the PU obtains from its spectrum asset. In section 5, we will illustrate the effect of CR SU activity on the mean and variance of active connections, and revenue, for a single cell mobile network.

III. FLOOR PRICE OF COGNITIVE RADIO RIGHTS

The presence of CR SUs operating in the PUs spectrum will affect the mean \( E(r_{spectrum}) \) and variance \( \sigma(r_{spectrum}) \) of return on the spectrum asset \( \Phi_{spectrum} \). Let \( \beta_M \) be the factor by which the expected return on the spectrum asset \( E(r_{spectrum}) \) is modified as a consequence of CR SU operation, such that the expected PU return with CR SU activity is:

\[
E(r_{spectrum})^* = \beta_M E(r_{spectrum})
\]

\(^1\) While it is not practically possible to earn \( r_f \) in intervals of less than 1 day, we can obtain the per-period \( r_f \) by dividing the daily rate by 86,400/\( \Delta t \) if \( \Delta t \) is in seconds.
Similarly, let $\beta_V$ be the factor by which the variance of returns on the spectrum asset is modified. The variance of PU returns with CR SU activity will be:

$$\sigma^2(r_{\text{spectrum}}) = \beta_V \sigma^2(r_{\text{spectrum}})$$

(8)

Results presented in [11] - [14] suggest that with CR SU activity, $E(r_n)$ will decrease, hence by eqn (4), $\beta_M < 1$; that is, CR activity will reduce the expected return on the spectrum asset. The parameter $\beta_V$ depends on the statistical characteristics of the PU traffic and the effect of CR activity on PU call dropping. The Sharpe Ratio of the spectrum asset with CR SUs present becomes:

$$S^*_\text{spectrum} = \frac{E(r_{\text{spectrum}})^* - r_f}{\sigma(r_{\text{spectrum}})^*}$$

(9)

The reward-to-variability graph (figure 1) illustrates the effect of the change in mean and variance of return on the Sharpe Ratio of the spectrum asset $\Phi_{\text{spectrum}}$. The gradient of the solid line is equal to the Sharpe Ratio $S_{\text{spectrum}}$ of the spectrum asset. All investments with mean and variance of returns ($\sigma(r)$), $E(r)$ on the solid line will have the same Sharpe Ratio and are equally desirable from a reward-to-variability point of view. The assets affected by a change in mean and variance of returns (due to CR SU activity) may fall below the solid line, and have a lower Sharpe Ratio $S'_{\text{spectrum}}$, making them less desirable investments from a reward-to-variability point of view.

Under the assumption that a PU finds the degradation of the Sharpe Ratio of its spectrum asset undesirable, the SU needs to compensate the PU such that the Sharpe Ratio of its spectrum asset is maintained. This can be achieved if the CR SU pays a coupon $c$ to the PU for the right to cognitively access the spectrum. If a coupon $c$ is added to the PU returns on the spectrum asset with CR operation $E(r_{\text{spectrum}})^*$, then the return will become $E(r_{\text{spectrum}})^* + c$ and the Sharpe Ratio becomes:

$$S^C_{\text{spectrum}} = \frac{E(r_{\text{spectrum}})^* + c - r_f}{\sigma(r_{\text{spectrum}})^*}$$

(10)

In figure 1, the coupon $c$ is the difference between the expected return of an asset on with the Sharpe Ratio $S_{\text{spectrum}}$ and the expected return of the asset whose Sharpe Ratio falls as a result of CR activity. To find the required coupon $c$ to maintain the Sharpe Ratio of the spectrum asset at its original value, we equate (3) and (10) and solve for $c$:

$$c = \frac{E(r_{\text{spectrum}})^* - r_f}{\sigma(r_{\text{spectrum}})^*} - E(r_{\text{spectrum}})^* + r_f$$

(11)

Substituting:

$$E(r_{\text{spectrum}})^* = \beta_M E(r_{\text{spectrum}})$$

(12)

$$\sigma^2(r_{\text{spectrum}})^* = \beta_V \sigma^2(r_{\text{spectrum}})$$

(13)

and simplifying, we obtain:

$$c = (\sqrt{\beta_V} - \beta_M) r_f (1 - \sqrt{\beta_V} r_f)$$

(14)

Obviously $c$ is a percentage of the up-front cost of the spectrum asset $K$, and the actual amount that will be paid is $cK$. In section 5 we will illustrate the impact of CR activity, specifically the effect of call dropping due to CR operation, on the factors $\beta_M$ and $\beta_V$.

The inclusion of the coupon $c$ ensures that the Sharpe Ratio of the PU spectrum asset remains unchanged when CR SUs are operating in the PU’s spectrum. From a reward-to-variability of returns point of view, the PU will not be biased to hold either asset: the exclusive spectrum asset $\Phi_{\text{spectrum}}$ or the spectrum asset with CR rights $\Phi^C_{\text{spectrum}}$. The coupon $c$ is the cash payment that the PU should obtain from the SU to grant CR rights for the next period $\Delta t$.

As the Sharpe Ratio of the portfolio $\Phi^C_{\text{spectrum}}$ is equal to that of the exclusive spectrum asset $\Phi_{\text{spectrum}}$, it can be modeled as a portfolio with capital divided between $y$ of the exclusive spectrum asset $\Phi_{\text{spectrum}}$ and $(1-y)$ of the risk-free asset [20]. The return on this portfolio is:

$$y E(r_{\text{spectrum}})^* + (1 - y) r_f = E(r_{\text{spectrum}})^* + c$$

(15)

Substituting (14) for $c$ and equating the coefficients of $E(r_{\text{spectrum}})^*$ and $r_f$ we obtain:

$$y = \sqrt{\beta_V}$$

(16)
Therefore, the effect for the PU of receiving a coupon $c$ in addition to reduced returns $E(r_{\text{spectrum}})$ for a timeslot, is the same as decreasing the leverage of the original investment in the spectrum asset $\Phi_{\text{spectrum}}$ by $1 - \sqrt{\beta}$ if $\beta < 1$, or increasing the leverage if $\beta > 1$ for that timeslot (Figure 2). (Decreasing leverage implies selling $1 - y$ of the risky asset and investing the proceeds in the risk-free asset; conversely increasing leverage involves borrowing additional capital at the risk free rate and buying $1 - y$ of the risky asset).

IV. SYSTEM MODEL

In this section we will define the example PU and SU communication system to illustrate the operation of the Sharpe ratio based CR rights valuation method. While we use a single cell model as an illustrative example, using appropriate traffic modeling techniques, our valuation method can be expanded to groups of cells or even entire mobile networks. We model the PU as a single cell mobile provider, with call arrivals described by a stationary Poisson process with rate $\lambda$ per timeslot and stationary exponentially distributed call durations with mean $\mu$. Therefore, the number of simultaneous active calls is independent of the service distribution $G$ and is given by [22]:

$$p_n = \frac{\lambda^n}{n!} \left( \sum_{i=0}^{\infty} \frac{\lambda^i}{i!} \right)^{-1}$$  \hspace{1cm} (17)

The CR SU network is located in the same region as the PU mobile cell (Figure 3), and utilizes the PUs spectrum. The SU operates under a PU QoS degradation constraint [11], [12] and [14]. Under an opportunistic access scheme, PU QoS degradation may occur if SUs commit sensing error (Figure 4a). Under a spectrum sharing scenario, PU QoS degradation will occur if SUs exceed a transmission power threshold [13]. In the case of the mobile cell, QoS degradation will be manifest by call dropping under the opportunistic access and spectrum sharing scenario, therefore our analysis is not restricted to either type of CR access scheme.

We assume that PU QoS degradation is manifest by call dropping; the probability of each active call being dropped being $P_{\text{drop}}$ in each timeslot. The relationship between the characteristics of the CR operation and $P_{\text{drop}}$ is beyond the scope of this paper, but under the opportunistic access scheme, $P_{\text{drop}}$ will be a function of PU collision probability, which is derived for various CR access schemes in [11], [12] and [13]. Under the spectrum sharing scenario, a derivation of PU outage probability is presented in [13]. We assume that calls can be dropped at the beginning of each timeslot, and if a call is dropped then no further revenue is earned from that call.

V. EFFECT OF CR ACTIVITY ON THE SPECTRUM ASSET MEAN AND VARIANCE OF RETURN

If the PU has exclusive rights to the spectrum (no CR activity), the mean number of active calls in each timeslot is given by:

$$E(n) = \sum_{j=0}^{C} j p_j$$  \hspace{1cm} (18)

If the call capacity $C$ is large compared to the offered load $\rho$, eqn (17) can be approximated as:

$$p_n = \frac{\rho^n e^{-\rho}}{n!}$$  \hspace{1cm} (19)

Therefore, the number of simultaneous calls $n$ in each timeslot is approximately Poisson distributed with $E(n) = \sigma^2(n) = \rho$, with the number of events in each timeslot being IID. The mean and variance of the return on the spectrum asset will be:

$$E(r_{\text{spectrum}}) = \frac{R}{K} \rho$$  \hspace{1cm} (20)

$$\sigma^2(r_{\text{spectrum}}) = \frac{R^2}{K} \rho$$  \hspace{1cm} (21)

And the single period Sharpe ratio will be:

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Figure 3: The example PU and SU model

Figure 4a: PU QoS degradation due to SU sensing error

Figure 4b: PU QoS degradation due to unsynchronized PU and SU sensing and transmission slots
Eqn (6) [21] can be applied to find the q-period Sharpe ratio from the single period Sharpe ratio, therefore in this example, it is sufficient to analyze the single timeslot mean and variance of active calls (and revenue).

With CR SUs operating in the PUs spectrum, the PU may suffer degraded QoS. In this example, we assume that each PU active call has a probability of being dropped \( P_{\text{drop}} \) at the beginning of each timeslot as a result of CR activity, and that a dropped call will not earn any further revenue. A simulation was performed to illustrate the effect of a call dropping probability \( P_{\text{drop}} \) on the mean and variance of active calls (and revenue) per timeslot. The average arrival rate \( \lambda \) was set to 17 calls/min (0.283 calls/sec) and the average call duration \( \mu \) was set to 3 min (180 sec) giving an offered load \( \rho \) of approximately 51 Erlangs. The simulation was carried out for approximately 51 Erlangs. The simulation was carried out for 86400 time-steps (simulating 1 day of mobile traffic, with each time step representing 1 second), and repeated 20 times, providing 20 samples of the mean and variance of active calls per timeslot. The effect on the mean and variance of active calls per timeslot is shown in figure 5, with the error bars indicating the standard deviation of the sample means and variances across the 20 trials.

Empirically, it seems that the distribution of active calls remains Poisson, as the mean and variance of active calls are approximately equal for all of the tested \( P_{\text{drop}} \) values. To test this further, we applied the Chi-Squared Goodness-of-Fit test to our results and failed to reject the null hypothesis \( H_0 \) at the 5% level, that the number of active calls \( n \) per timeslot with \( P_{\text{drop}} = \{0, 0.001, 0.002, \ldots, 0.1\} \) is Poisson distributed with mean and variance equal to the sample mean \( \lambda_{(\text{Sample})} \) and variances across the 20 trials.

This is also evident by graphically comparing the sample CDF with the Poisson CDF that has the same mean and variance as the simulated samples (Figure 6).

Therefore, we can assume that the number of events in each timeslot remains IID, and the per-second Sharpe ratio analysis is sufficient. In this example, the factor modifying the mean and variance of the number of active calls (and the return on the spectrum asset) will be approximately equal: \( \beta_i \approx \beta_y = \beta \). For the above values of \( P_{\text{drop}} \) results for \( \beta \) are shown in table 1, with \( \beta \) calculated as the mean of \( \beta_i \) and \( \beta_y \). As the mean and variance of the number of active calls per timeslot is equal in this example, the effect of CR activity on the Sharpe ratio of the spectrum asset can be illustrated by a parabola \( A \) on the reward-to-variability graph (Figure 7). As the call dropping probability \( P_{\text{drop}} \) increases (and \( \beta \) decreases), the mean-variance characteristic of the spectrum asset will move further down along the parabola \( A \).

### Table I. Factors modifying mean and variance of active calls for various \( \ P_{\text{drop}} \) levels

<table>
<thead>
<tr>
<th>( P_{\text{drop}} )</th>
<th>Sample mean of active calls/timeslot</th>
<th>Sample variance of active calls/timeslot</th>
<th>( \beta_i )</th>
<th>( \beta_y )</th>
<th>( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>51.045</td>
<td>53.0799</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
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<tr>
<td>0.001</td>
<td>43.3193</td>
<td>45.0445</td>
<td>0.848649</td>
<td>0.848617</td>
<td>0.848633</td>
</tr>
<tr>
<td>0.002</td>
<td>37.4614</td>
<td>38.4745</td>
<td>0.73389</td>
<td>0.724841</td>
<td>0.729656</td>
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<td>0.003</td>
<td>33.1719</td>
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<td>0.630548</td>
<td>0.640202</td>
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<td>0.344533</td>
<td>0.349266</td>
</tr>
</tbody>
</table>

In this example the effect on the Sharpe ratio of the spectrum asset is described by:

\[
S_{\text{specrum}} = \frac{R - r_f}{\sqrt{\sigma^2}}
\]
The floor price of CR rights given a modifying factor $\beta$ arising from a dropping probability $P_{\text{drop}}$ will be:

$$c(\beta) = (\sqrt{\beta} - \beta)E(\text{spectrum}) + (1 - \sqrt{\beta})r_f$$

And the Sharpe ratio of the spectrum asset with the coupon $c$ included will be:

$$S^c_{\text{spectrum}}(\beta) = \frac{R}{K} \beta \rho + c(\beta)$$

For $R = 0.005/\text{call/sec}$, $K = 40$ million, and $r_f = 5\%$ annualized (1.5855$\times 10^{-7}\%$ per second), figure 8 illustrates the per-second (annualized) sample mean spectrum asset returns with varying levels of CR activity (characterized by the $P_{\text{drop}}$ parameter) compared with the per-second (annualized) sample mean returns when the coupon is included and paid every second by the SU.

The per-second Sharpe ratio of on the spectrum asset with varying levels of CR activity is shown in figure 9. This is compared to both the theoretical (from eqn (25)) and simulated per-second Sharpe ratio of the spectrum asset when the coupon is included and paid every second. Note that in this example, while the coupon paid by the SU to the PU is less than the reduction in the mean return on the spectrum asset, its inclusion restores the Sharpe ratio of the spectrum asset to its original value (with no CR activity), hence the quality of the Spectrum asset as an investment is not degraded with CR activity. This is an example of the ‘decreased leverage’ effect described in section 3 and by figure 2.

For the purpose of illustrating the valuation method, we have assumed that the call arrival, duration and dropping processes are stationary. In reality, the distributions of these parameters may vary with time. In that case the analysis of the per-period mean and variance of active calls is still possible, but the non-IID methods of estimating q-period Sharpe ratios presented in [21] will need to be utilized. Practically, the parameter $\rho$ will have to be estimated from historical call data in both the stationary and time-varying case. Also in this example the effect of CR SU activity decreased the variance as well as the mean of the return on the spectrum asset, and the coupon payment required from the SU was actually less than the reduction in the mean return. However, as figure 2 illustrates, scenarios where the variance is increased ($\beta > 1$), the PU would have to be compensated more than the reduction in mean spectrum asset return. To extend this analysis to a multi-cell or network scenario, larger scale traffic modeling is required to find the mean $E(n)$ and variance $\sigma^2(n)$ of active calls (and hence revenue).

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VI. CONCLUSION

We have presented a method for finding the floor price that a Primary User license holder should charge a Secondary User to access its spectrum cognitively, while preserving the reward-to-variability ratio of its investment in the spectrum license, hence its economic welfare. The example presented illustrates the effectiveness of such a pricing method in maintaining the Sharpe ratio of the spectrum asset with varying levels Cognitive Radio activity. With appropriate traffic modeling techniques, our valuation method can be applied to multi-cell or mobile network scenarios. Such an approach could be used in conjunction with existing market and auction based proposals for pricing Cognitive Radio rights to ensure that the risk-reward characteristics of the investments that PUs have made in radio spectrum are not degraded. Further development of this approach would include explicit models for PU QoS deterioration, as well as the incorporation of suitable revenue collection mechanisms at the PU and SU systems.

REFERENCES