Subcarrier Allocation and Bit Loading Algorithms for OFDMA-Based Wireless Networks

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Abstract—Orthogonal Frequency Division Multiple Access (OFDMA) is an emerging multiple access technology. In this paper, we consider OFDMA in the context of fixed wireless networks. This paper addresses the problem of assigning subcarriers and bits to point-to-point wireless links in the presence of cochannel interference and Rayleigh fading. The objective is to minimize the total transmitted power over the entire network while satisfying the data rate requirement of each link. We formulate this problem as a constrained optimization problem and present centralized algorithms. The simulation results show that our approach results in an efficient assignment of subcarriers and transmitter power levels in terms of the energy required for transmitting each bit of information. However, centralized algorithms require knowledge of the entire network topology and channel characteristics of every link. In a practical scenario, that would not be the situation and there is a need for distributed rate allocation algorithms. To address this need, we also present a distributed algorithm for allocating subcarriers and bits in order to satisfy the rate requirements of the links.

Index Terms—Resource allocation, joint rate allocation and power control and scheduling, orthogonal frequency division multiplexing (OFDM).

1 Introduction

Scarcity of the wireless spectrum requires the design of efficient medium access control (MAC) schemes. The nature of wireless links is fundamentally different from their wired counterparts. In wired networks, a link exists between two nodes if they are physically connected using a communication cable. These links have fixed capacity. However, the quality or even the very existence of wireless links depends on a number of factors such as the target bit error rate (BER), signal to interference and noise ratio (SINR), the modulation/coding scheme used, etc. Thus, wireless links can be thought of as being in one of several "states" depending on the above parameters, with each state being characterized by a certain performance level. This has two important consequences. First, the capacity of a wireless link is variable and depends on the states of other links in the network as well. Second, even in the absence of mobility, the network topology is not well defined. Both of these consequences have a profound impact on several layers of the protocol stack. For example, spatial reuse of channels, performed to increase the network capacity and conserve scarce network resources, should consider physical layer characteristics. Also, the medium access control (MAC) scheme needs to be sensitive to the channel quality experienced by different transmitters and multiuser diversity is exploited. The goal of our work is to study the joint optimization of the physical and MAC layers. We introduce a common framework for joint power control, rate control, and scheduling. We formulate a generic optimization problem for the same and show that problems such as classical power control and joint scheduling and power control are special cases. However, we will focus on subcarrier allocation and bit loading for Orthogonal Frequency Division Multiple Access (OFDMA).

Orthogonal Frequency Division Multiplexing (OFDM) is a spectrally efficient digital modulation technique. The spectrum of interest is divided into a number of parallel orthogonal narrowband subchannels (known as subcarriers). The purpose is to convert a frequency-selective fading channel into several flat-fading subchannels. Data symbols are transmitted in parallel on each subcarrier with low symbol rates, as a result of which intersymbol interference (ISI) is reduced. When different subcarriers experience different channel gains and interference levels, adaptive bit loading [1] is performed to enhance the throughput for a given transmission power limit. Fading parameters for wireless links at different locations are independent. Fig. 1 illustrates the basic operation of an OFDM communication system. The band of interest is split into N orthogonal subcarriers. In most systems, each subcarrier is modulated with M-ary Quadrature Amplitude Modulation (QAM). The collection of symbols on the N subcarriers is known as an OFDM symbol.

OFDMA exploits location-dependent channel diversity and frequency selectivity. Adaptive subcarrier allocation combined with adaptive bit loading and power control is performed. Adaptive modulation on each subcarrier exploits the channel diversity while power control is

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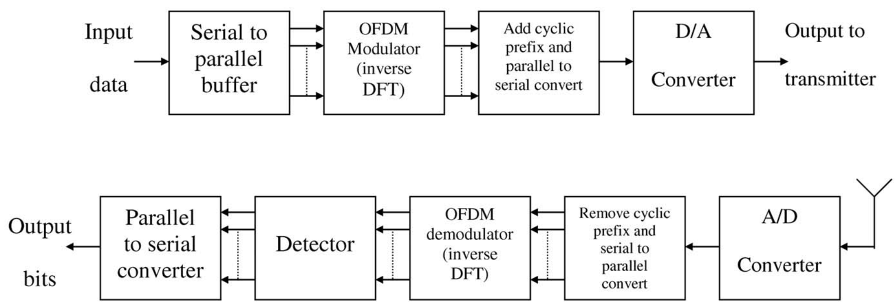

Fig. 1. OFDM communication system [13].

necessary to maintain the required signal to interference and noise ratio (SINR) on a wireless link for it to be able to support a given data rate on a given subcarrier. Spectrum reuse not only increases the capacity of wireless networks, but also leads to interference. Hence, power control is necessary to mitigate the effects of the interference.

1.1 Related Work

Adaptive modulation with power control [2] has been studied in the context of cellular systems. It was shown that adaptive modulation results in higher throughputs compared to classical power control [14]. In [3], the authors present carrier assignment algorithms for broadband wireless networks by trying to assign carriers to users to minimize the number of channels (in this case, time slots) used by each user. In [4] and [5], the authors present bit loading and subcarrier allocation algorithms for multiuser OFDM. They utilize the fact that users in different locations experience different fading characteristics and try to minimize the transmitted power while meeting their rate requirements. However, their results are applicable for a single cell system in which there is no interference. The authors in [6] deal with adaptive modulation and power allocation in a multicell OFDM system. In their system, all users share all the subcarriers on a TDMA basis. In [7] and [8], the authors address the problem of joint scheduling and power control. The authors of [8] perform the scheduling by finding feasible cochannel sets of links, or transmitter-receiver pairs. For the transmissions to be feasible, the SINR at each receiver must exceed the corresponding threshold and power control is performed.

The work most closely related to ours is by Cioffi et al. [9], [10], [11], [12]. In [9], they present the iterative water-pouring (IWF) scheme, which maximizes the aggregate data rates of mutually interfering DSL modems. A game-theoretic approach is used and the Nash Equilibrium is characterized for the two-user case. However, IWF is not optimal and, in [10], another suboptimal method, known as iterative constant-power transmission (ICP), is presented. ICP is a variant of IWF in which each user transmits with a constant power over selected frequency tones. Both IWF and ICP are compared to the optimal solution for a two-user case. In [11] and [12], algorithms very similar to one of our algorithms that we have in our preliminary work [23] were developed in parallel. The objective is to minimize the total power consumption while satisfying the data rate requirement of each link. A greedy heuristic is employed. The heuristics in [11] and [12] are shown to be superior to IWF. However, we shall later show in Section 3 that, under certain circumstances, greedy heuristics fail to find a feasible solution even when the problem is feasible. We present another algorithm that is more successful in finding feasible solutions. In addition, we present a distributed algorithm and compare its performance with various centralized schemes. The distributed algorithm only requires local knowledge, i.e., the SINRs at the receiver for each subcarrier, while the centralized algorithms require global knowledge of all channel gains. Despite this constraint, we show that the distributed algorithm performs very well and the performance penalty is not too high.

We now briefly review some work on power control. In [18], the authors describe a plethora of power control and admission control algorithms. They derive a number of interesting mathematical properties of these algorithms. The most important contribution of [18] is the introduction of the voluntary dropout scheme to the Dynamic Power and Channel allocation with Active Link Protection (DPC/ALP), which was presented in [19]. This scheme uses DPC/ALP as the underlying channel access scheme. As the DPC/ALP iterative algorithm evolves, all the SINRs of all new links are guaranteed to be nondecreasing. However, it may not reach the desired level. In this case, the new links may voluntarily drop out or be forced out. The criteria for dropping out could be based on a time-out or on a probabilistic basis. We adopt similar techniques in our distributed algorithm.

1.2 Paper Contributions and Overview

We begin by introducing a common framework for joint power control, rate control, and scheduling. We formulate a generic optimization problem for the same and show that problems such as classical power control and joint scheduling and power control are special cases. In this paper, we focus on fixed infrastructureless wireless networks that employ OFDMA as the MAC scheme. We assume an idealized time and frequency synchronous system. We address the problem of subcarrier, bit, and power assignment for point-to-point fixed wireless links in the presence of cochannel interference and Rayleigh fading due to multipath propagation. The important issues to be considered are location-dependent fading characteristics and power control. Our objective is to minimize the total transmitted power over all links while maintaining the data rates on each link. The authors of [17] provide computationally efficient algorithms for subcarrier and bit allocation in a single cell scenario. In our situation, we have additional constraints due to interfering links. We first present centralized algorithms for subcarrier and bit allocation in the presence of interfering links. However, centralized algorithms require knowledge of the entire network topology and channel characteristics of every link. In a practical scenario, that would not be the situation and there is a need for distributed rate allocation algorithms. To address this need, we also present a distributed algorithm for allocating subcarriers and bits in order to satisfy the rate requirements of the links.

To the best of our knowledge, the problem of distributed subcarrier and bit allocation with power control for ad hoc networks has not been addressed in prior literature. The centralized algorithms were first presented in our conference paper [23]. Based on our centralized algorithms, we obtain a distributed algorithm and discuss the trade offs involved. The paper is organized as follows: In Section 2, we present the system model and formulate our problem. In Section 3, we motivate and present our centralized rate allocation algorithms. In Section 4, we present our distributed algorithm, and performance evaluation is presented in Section 5. We discuss some practical issues in Section 6, and Section 7 concludes this study.

2 System Model and Problem Formulation

We consider an OFDMA-based wireless network with no base station infrastructure. The network consists of point-to-point wireless links (sets of transmitter-receiver pairs), each with a certain data rate requirement R_i. Let the spectrum of interest be divided into N subcarriers. There are a total of M links in the network. We consider an environment in which, for a given link, by appropriate choice of subcarrier spacing and symbol rate, the channel for each subcarrier experiences flat fading. Here, P_i^c is the power transmitted by transmitter i on subcarrier c and I_i^c is the corresponding interference power. P_i^c is equal to zero if link i does not use subcarrier c. Let G_{ij}^c be the gain from the transmitter of link j to the receiver of link i for subcarrier c. Here, G_{ij}^c includes the path loss and the effect of frequency-selective fading. In our numerical results, we select parameters similar to that of the IEEE 802.11a physical layer [26]. Note that we assume an idealized time and frequency synchronous system. Frequency and timing synchronization are issues under investigation and beyond the scope of this paper. We note that our algorithms are general and designed to work for any channel model provided the gain parameters are known. For the distributed algorithm, the transmitter only needs to know the SINR at the corresponding receiver.

The SINR of link i for subcarrier c is given by:

$$\gamma_i^c = \frac{G_{ii}^c P_i^c}{N_i + \sum\limits_{j=1, j\neq i}^{M} G_{ij}^c P_j^c}. \tag{1}$$

Let b_i^c be the number of bits transmitted by link i on subcarrier c. When M-ary quadrature amplitude modulation (M-QAM) [13] is used, the corresponding SINR threshold is:

$$\overline{\gamma_i^c} = [Q^{-1}(BER/4)]^2 (2^{b_i^c} - 1)/3, \tag{2}$$

where BER is the target bit error rate and Q(.) is the Gaussian tail function given by

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-t^2/2} dt. \tag{3}$$

We now formally state our problem. The data rate R_i can be expressed as:

$$R_i = \sum_{c=1}^{N} b_i^c. \tag{4}$$

When K links $(i_1, i_2, \dots, i_K)$ are transmitting on subcarrier c, we require that

$$\gamma_{i_m}^c \geq \overline{\gamma_{i_m}^c}, m = 1, 2, \dots, K. \tag{5}$$

In matrix form, these conditions can be written as:

$$(I - F^c) P^c \geq U^c, c = 1, 2, \dots, N, \tag{6}$$

where $\boldsymbol{P}^c = \left(P_{i_1}^c, P_{i_2}^c, \dots, P_{i_K}^c\right)^T$ is the column vector of all strictly positive transmitter powers for subcarrier c and $\boldsymbol{I}$ is the identity matrix and the notation $\boldsymbol{x}^T$ denotes the transpose of the vector $\boldsymbol{x}$. $\boldsymbol{U}^c$ is the column vector of thermal noise powers scaled by SINR thresholds and link path loss factors. These were obtained in [14].

$$U^c = \left(\frac{\overline{\gamma_{i_1}^c} N_{i_1}}{G_{i_1 i_1}^c}, \frac{\overline{\gamma_{i_2}^c} N_{i_2}}{G_{i_2 i_2}^c}, \dots, \frac{\overline{\gamma_{i_K}^c} N_{i_K}}{G_{i_K i_K}^c}\right)^T. \tag{7}$$

$\boldsymbol{F}^c$ has the following entries:

$$F_{pq}^c = \begin{cases} 0 & \textit{if } p = q \\ \frac{\overline{\gamma_{i_p}^c} G_{i_p i_q}^c}{G_{i_p i_p}^c} & \textit{if } p \neq q. \end{cases} \tag{8}$$

It was shown in [14] that a positive solution for $\boldsymbol{P}^c$ exists if the maximum eigenvalue of $\boldsymbol{F}^c$ is less than 1. Otherwise, the set of SINR thresholds, or, in other words, the set of modulation levels used by all the links on subcarrier c, is not feasible.

Our objective is to minimize the total transmission power over all the links under the constraints that the data rates for all links must be achieved and the SINR for every link on each subcarrier that it uses should exceed the corresponding threshold. The goal is to find an assignment of subcarriers, bits, and power, or, in other words, find b_i^c and P_i^c for every link i and subcarrier c. We note that the independent variables here are b_i^c for $i = 1, \dots, M$ and $c = 1, \dots, N$.

Lemma 1. *P_i^c is a function of $\{b_1^c, b_2^c, \dots, b_M^c\}$ for $i = 1, \dots, M$ and $c = 1, \dots, N$ if $\lambda_{\max}^c < 1$, where $\lambda_{\max}^c$ is the maximum eigenvalue of the matrix $\boldsymbol{F}^c$ as defined in (8).*

Proof. Based on the Perron-Frobenius Theorem [19], [24], a component wise nonnegative solution for $\boldsymbol{P}^c = [P_1^c, P_2^c, \dots, P_M^c]^T$ exists only when $\lambda_{\max}^c < 1$. In that case, the Pareto optimal solution is

$$P^c = \left[I + \sum_{k=1}^{\infty} (F^c)^k\right] U^c.$$

Thus, we see that each component P_i^c is a function of $\{b_1^c, b_2^c, \ldots, b_M^c\}$. □

In other words, the power transmitted by a link on a particular channel depends not only on its own bit rate in that channel, but also of all other links sharing that channel. If a particular link i does not use subcarrier c, then b_i^c and, hence, P_i^c are zero. The optimization problem is formally stated as:

$$\begin{aligned} & Minimize \quad \sum_{i=1}^{M}\sum_{c=1}^{N} P_i^c \\ & Subject\ to: \\ & R_i = \sum_{c=1}^{N} b_i^c \quad i = 1,2,...,M \\ & \sum_{c=1}^{N} P_i^c \le P_{\max} \quad i = 1,2,...,M \\ & b_i^c \ge 0 \\ & \forall\, i = 1,2,...,M \ and \ \forall\, c = 1,2,...,N. \end{aligned} \tag{9}$$

This is a constrained nonlinear optimization problem [15]. We also bear in mind the fact that b_i^c take only integer values $\in \{0,1,2,\ldots,b_{max}\}$, where b_{max} is the maximum modulation level used. Thus, this is also a combinatorial optimization problem. Finding the global minimum requires an exhaustive search over all possible assignments of subcarriers to links. For a particular link i with rate requirement R_i, the number of ways in which R_i bits can be distributed over N subcarriers is N^{R_i}. Since there are a total of M links, the number of possible assignments is:

$$\prod_{i=1}^{M} N^{R_i} = N^{\sum_{i=1}^{M} R_i}.$$

This problem is apparently intractable due to the exponential dependence on M. As a consequence, we have to resort to suboptimal heuristics for a computationally manageable solution.

The above problem formulation is very general in the sense that the rate, schedule, and power levels need to be selected. When each b_i^c takes only binary values $\in \{0, K\}$ for some integer K, the problem reduces to joint power control and scheduling [7], [8], [22]. Now, R_i represents the number of times a link needs to be scheduled and this depends on the traffic. Clearly, the problem is still combinatorial. When $N = 1$, i.e., all links share the same channel, it reduces to the classical power control problem, and the Pareto Optimal solution, when it exists (based on the eigenvalue condition), is unique. The objective function can be thought of as the "cost" of rate allocation and scheduling over the entire network. Thus, for the same data rate requirements, lower aggregate transmission power implies more efficient resource allocation.

3 Centralized Rate Allocation Algorithms

In this section, we present our algorithms that meet the rate requirements of the links while trying to minimize the total transmitted power.

3.1 Motivation and Description of Algorithm

The aim is to assign a set of subcarriers to each link and load bits on each assigned subcarrier. The factors to be considered are the frequency-dependent path loss parameters and the interference due to other links that have already been assigned those subcarriers. Alternatively, this can be looked upon as identification of sets of links that share each subcarrier. When a new link is assigned a certain subcarrier with a particular bit loading, the SINR thresholds for itself as well as for all other links already using that subcarrier must be maintained. Power control must be performed to maintain the required thresholds. The new set of links sharing the same subcarrier c is feasible if the maximum eigenvalue of the F^c matrix is less than one for every c. If feasible, the new transmitter power values are obtained from (6) and this leads to an increase in the total transmitted power. Different combinations of links and subcarriers will lead to different increments in the total transmitted power. This is heavily dependent on the *order* in which subcarriers are assigned to links.

The problem amounts to finding an appropriate mapping of the bits of each link to different subcarriers such that all SINR thresholds are maintained and the total transmitted power, after power control, is minimum. We present an algorithm that loads one bit of a link onto a subcarrier at each step such that the incremental transmitted power after performing power control is minimum. This process is continued until either all bits from each link have been successfully loaded onto some subcarrier or it is not feasible to load any more bits. Let $\triangle P(i, c, b_i^c)$ be the total increase in transmitter power over all links when one bit of link i is loaded on subcarrier c when b_i^c bits of link i are already loaded on subcarrier c. If link i is the only link that is assigned to subcarrier c, i.e., there are no interfering links, then $\triangle P(i, c, b_i^c)$ is given by

$$\Delta P(i,c,b_i^c) = \Gamma N_i \left(2^{b_i^c+1} - 2^{b_i^c}\right), \tag{10}$$

where Γ is as given in (9) and N_i is the noise power. However, in the presence of interfering links, $\triangle P(i, c, b_i^c)$ is obtained by the difference between the total transmitted power on subcarrier c before and after the addition of one bit and power control being performed. For a given set of modulation levels on subcarrier c, the corresponding transmitter powers are obtained from (6). If it is not feasible to add any more bits of link i on subcarrier c, we mark the tuple (i, c) as not feasible. Fig. 2 outlines the steps of our Minimum Incremental Power Algorithm (MIPA).

We note that it is required to check the eigenvalue feasibility condition only when there are interfering links and not otherwise. The output of our algorithm are b_i^c and P_i^c for every link i and subcarrier c. Clearly, this is a suboptimal, greedy heuristic.

3.2 Special Cases

We list below some special cases for which our algorithm reduces to solving other problems.

1. M = 1: rate assignment for a single link.
   In this case, there are no interfering links and the problem reduces to the equivalent problem of

```
Minimum Incremental Power Algorithm (MIPA)

Do while (rates not met) and (at least one (i,c) feasible) {
    For each link i from 1 through M {
      If rate requirement of i is not yet satisfied {
        For each subcarrier c from 1 to N {
          If the tuple (i,c) is feasible {
            Check if it is feasible to add one bit to b_i^c
            If (i,c) is still feasible {
                Calculate ΔP(i,c,b_i^c)
                If any power constraint is violated
                    Mark (i,c) as infeasible
                    Set ΔP(i,c,b_i^c) = ∞
            }
          }
        }
      }
    }
    Select (J,C) = argmin_{i,c} ΔP(i,c,b_i^c)
    Load one bit of link J onto subcarrier C
    Perform power control and update transmitter powers
}
```

Fig. 2. Minimum Incremental Power Algorithm (MIPA).

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Minimum Subcarrier Allocation Algorithm (MSAA)

Do while (rates not met) and (at least one (i,c) feasible) {
    For each link i from 1 through M {
      If rate requirement of i is not yet satisfied {
        For each subcarrier c from 1 to N {
          Initialize b = b_max
          Do while b > 0 {
            Check if it is feasible to load b bits of link i at
            subcarrier c
            If feasible and no power constraint violated,
            calculate ΔP
            Else b = b – 1
          }
        }
        Select subcarrier C with highest b
        If multiple subcarriers found
            Choose the one with lowest ΔP
        Load min(b, remaining_rate(i)) bits on C
        Perform power control and update transmitter powers
      }
    }
```

Fig. 3. Minimum Subcarrier Allocation Algorithm (MSAA).

assigning subcarriers and bits for a single user scenario. For this situation, the greedy algorithm is optimal and reduces to single-user bit loading.

2. N = 1: transmission over a single channel.

   In this situation, all links are transmitting over the same channel. The problem now reduces to a simple power control problem. The solution exists if the highest eigenvalue of the F^c matrix is less than one. This solution is unique and optimal.

3.3 Alternate Heuristic

There are two conflicting issues while selecting subcarriers and loading bits. The transmitted power over a subcarrier increases exponentially with the number of bits that are loaded on it. This might prompt us to adopt a strategy to use low modulation levels on each subcarrier and spread out the transmitted power over a large number of subcarriers. However, this means that any subcarrier would be reused by many links and, to maintain the required SINR, all of them would have to increase their transmission powers. The algorithm we described above follows this approach.

An alternate strategy might be for each link to minimize the number of subcarriers it uses by using high modulation levels. Although the transmitted power over the selected subcarriers is high, the number of links reusing the same subcarriers is less. We present below an algorithm that tries to minimize the number of subcarriers that are used by a link. There is an upper limit, b_{max}, on the number of bits that can be loaded on each subcarrier. The basic idea is the following: Every link is serviced in a round-robin fashion. At each step, and for each link, we find the subcarriers on which maximum number of bits can be loaded based on power control feasibility for that subcarrier. If multiple subcarriers are found, we select the one that results in the least incremental total transmission power (after power control is performed) over all the links. This is repeated until either all rate requirements are satisfied or it is not feasible to add any more bits. Fig. 3 outlines the steps of our Minimum Subcarrier Allocation Algorithm (MSAA).

3.4 Graph-Based Approach

In [8], the authors address the problem of joint scheduling and power control in wireless networks. In their model, they consider point-to-point links with fixed data rates (no rate control) in a TDMA MAC with a given frame length. Their goal is to schedule all the links and maximize the spatial reuse of channels. As we have noted before, this is a special case of the problem we are trying to address. The main distinction is that, while they consider a single-rate, single-channel MAC, we consider a multirate multichannel MAC. However, it is possible to adapt their Power Control and Scheduling Algorithm (PCSA) and apply it to our problem. We shall then compare its performance with our centralized and distributed algorithms (described in Section 4).

They introduce the concept of interference graphs. The links in the network form the vertices of the graph. An edge exists between any two vertices if the corresponding links cannot find a feasible transmission power vector for a particular channel. In a single-rate system, the feasibility can be easily determined using the eigenvalue condition. Since the data rate is fixed, the SINR thresholds are known. The interference graph captures the pairwise interaction between links and not the effects of aggregate interference on the SINRs. In [8], a few observations are made about the properties of the interference graph. Forming of the interference graph in our multirate system is not trivial as there are several ways in which the total data rate requirement of a link can be split over multiple channels. Each link can thus be thought of as multiple "sublinks" and each sublink then forms the vertices of the interference graph. The feasibility must be checked for between sublinks. For example, links i and j may be feasible when both use BPSK, but infeasible when i uses BPSK and j uses QPSK. We can now apply PCSA to do the scheduling. Note that, unlike [8], we consider frequency-selective fading channels, so we have to construct an interference graph per channel. This is because two sublinks that may be infeasible in one channel may be feasible in another channel.

We now present the modified version of PCSA, adapted to solve our problem. The challenge lies in finding a good split. This is an extremely hard problem. We adopt the strategy of using small modulation levels and spreading the data rate over a large number of channels. This would imply smaller power levels per channel and higher spatial reuse. Before we describe the modified PCSA algorithm, we introduce the following notation to describe a sublink. Each link i, with total rate requirement R_i, is split into $X(i)$ sublinks denoted by (i, b_j), where

$$\sum_{j=1}^{X(i)} b_j = R_i.$$

$X(i)$ and b_j are given as follows:

$$X(i) = \begin{cases} R_i \text{ if } R_i \leq N \\ N \text{ if } R_i > N \end{cases}$$

and

$$b_j = \begin{cases} \left\lfloor \frac{R_i}{N} \right\rfloor & +1 \text{ if } 1 \leq j \leq (R_i \bmod N) \text{ and } R_i > N \\ \left\lfloor \frac{R_i}{N} \right\rfloor & \text{if } (R_i \bmod N) < j \leq X(i) \text{ and } R_i > N \\ 1 & \text{if } 1 \leq j \leq X(i) \text{ and } R_i \leq N, \end{cases}$$

where $\lfloor a \rfloor$ denotes the greatest integer less than or equal to a. The construction of the interference graphs is as follows:

Step 1. Let V denote the collection of all sublinks and E^c denote the edges for the graph H^c corresponding to channel c. Sublinks (i, b_j) and (k, b_l) are connected by an edge if:

a. $i \neq k$ and (i, b_j) and (k, b_l) are not feasible for $(j, l) \in \{1, 2, \ldots, X(i)\} \times \{1, 2, \ldots, X(k)\}$ or
b. $i = k$ and $j \neq l$ for all $j, l \in \{1, 2, \ldots, X(i)\}$, i.e., in any channel, only one of the sublinks of a given link can be scheduled.

Step 2. Repeat step 1 for all channels.

The final algorithm is as follows:

Step 1. Construct the interference graphs $H^c = (V, E^c)$ for all $c \in 1, 2, \ldots, N$.

Step 2. Start with $c = 1$.

Step 3. Find a maximal independent set of H^c using the Minimum Degree Greedy Algorithm [25].

Step 4. From the maximal set, find a feasible set of transmissions (S).

Step 5. Trim the interference graphs for all channels by removing S.

Step 6. Proceed to next channel—stop if all channels scheduled or all sublinks are scheduled. For details, the reader is urged to refer to [8].

4 Distributed Algorithm

In Section 2, we assumed complete knowledge, i.e., given a set of links with rate requirements and knowledge of channel gains for the entire network, we had to allocate bits and subcarriers. We then provided centralized algorithms to perform rate allocation. In this section, our aim is to develop a distributed algorithm for fulfilling the rate requirements of different links. As before, we consider an OFDMA-based network. The algorithm would be running independently on the transmitting node of each link with cooperation from the receiver. There would be no coordination with other links. In this case, the node that runs the algorithm has no knowledge of channel gains for the entire network—all it knows is the state of the channels at the receiver.

Every link periodically performs power control for every subcarrier that it uses. Time is divided into slots and every link updates its power at the end of each slot as follows:

$$P_i^c(k+1) = \frac{\overline{\gamma_i^c}}{\gamma_i^c} P_i^c(k), \tag{11}$$

where $P_i^c(k)$ is the power transmitted by link i on subcarrier c in time slot k and γ_i^c is the measured SINR at the receiver of link i. The corresponding SINR threshold $\overline{\gamma_i^c}$ is as given by (2). It was shown in [14] that the power update (11) converges to the Pareto optimal solution when the maximum eigenvalue of the $\boldsymbol{F}^c$ matrix described in Section 2 is less than 1 and the convergence is exponentially fast [27].

The objective is still to fulfill the rate requirements of each link and to expend minimum power in the process. The aim is now to make a judicious choice of subcarriers and load them appropriately, taking into account the gain parameters. Before delving into the details of our algorithm, we discuss some key issues that must be addressed.

4.1 Motivation and Justifications

All links are simultaneously attempting to schedule transmissions on different subcarriers. The allocation of power to different subcarriers must be globally feasible, i.e., it must yield a solution such that transmitted powers of all links are stable after power control updates. There is no means by which this can be determined a priori as the modulation levels used by the links on any subcarriers are not predecided. Since each link is unaware of the global network conditions, a commonly used technique is to transmit a probing signal and observe the effect it has on all cochannel links. After a few power control updates, it can be determined whether a particular SINR threshold is feasible on that link such that it yields a globally feasible solution. This technique was first proposed in [19] and is known as channel probing. The authors show that local admissibility is equivalent to global feasibility. The probing signal should be several orders of magnitude smaller than the transmission powers, otherwise, it may be strong enough to cause link powers to diverge.

While this technique is extremely useful for single carrier systems, it has several limitations when used for multicarrier systems. If all the subcarriers are probed simultaneously, i.e., if probing signals are sent over the entire spectrum, the time domain signal has a large peak to average power ratio (PAPR), which is undesirable. Also, in single carrier systems, the required SINR threshold is known, while, in multicarrier systems, it is not known before bits are loaded. This is because we do not have a priori knowledge of which subcarriers are going to be used and what the bit loading is going to be. If all subcarriers were to be probed one by one and not simultaneously, the allocation would take a long time. Thus, instead of probing the subcarriers one after the other, we could directly start loading bits and then check for feasibility.

The other important consideration is the time taken to allocate subcarriers and bits. A certain minimum time must be spent in order to check whether the power control updates are going to converge to a stable solution. On the other hand, spending too much time is undesirable since it increases the latency before data can be transmitted. Also, the channel conditions and interference conditions might change during that time so that existing bit and power allocations are no longer optimal. We must also keep in mind that the objective is to meet the rate requirement by spending as little power as possible. Thus, from among several feasible alternatives, the ones with less transmitter power should be selected.

We would also like to point out that it might not always be possible to find a feasible allocation that satisfies the total rate requirement of the link. As a consequence, whenever power control is performed, there should be some form of admission control. This problem has been addressed for single carrier systems. The DPC/ALP scheme with voluntary dropping out of links that are unable to meet their SINR thresholds is one approach [19]. This scheme protects links that have already been allocated a channel, but takes a long time to admit new links. If this were to be used for every subcarrier, the time taken would be prohibitively long. Another channel probing scheme [20], which is an extension of [19], requires that each channel be probed one after the other. The probing time is less but it does not protect active links. It is not a good idea to probe all subcarriers simultaneously due to the large PAPR and, as mentioned earlier, we are better off starting the bit allocation process directly rather than probing each subcarrier one after the other. Clearly, what we need is a method for solving our specific problem taking into account the considerations mentioned above.

4.2 Description of Algorithm

Drawing ideas from the MIPA and MSAA algorithms, we now try to adapt our centralized algorithms into a distributed form. Every new link has to select a set of subcarriers, load an "appropriate" number of bits, which will determine the SINR thresholds. The link then performs power control in order to meet the thresholds. The choice of subcarriers and bit allocation should be such that the link should expect to have the minimum possible transmission power to transmit the same number of bits. As we discussed earlier, bit and subcarrier allocation has certain latency.

We develop an algorithm in which, at each step, a link selects a particular subcarrier and loads one bit and then performs power control to try to achieve the corresponding SINR threshold. The criterion for selecting the subcarrier is the G_i^c/I_i^c factor—the subcarrier with the highest G_i^c/I_i^c factor *at that instant* is selected, where G_i^c and I_i^c are the channel gain and interference, respectively, for link i on subcarrier c. In other words, the link tries to select the subcarrier which is good for itself as well as has least interference. When the link starts transmitting power on the newly selected subcarrier, it increases the interference seen by existing links that use the same subcarrier. Due to power control, they will increase their transmission powers. This process continues and will converge if the feasibility condition is satisfied. If the assignment is feasible, the allocation was successful and the link selects the next subcarrier and repeats the process.

The question that now arises is what must be done if after a few power control updates (W slots), the power transmitted by the link on the selected subcarrier does not stabilize and is still increasing? We must therefore have some form of admission control. It is possible that the infeasibility in the channel being considered is due to multiple links trying to gain access to it simultaneously. If all of them drop out, the channel would be underutilized. Therefore, we adopt a probabilistic approach. Each link i drops out with a probability $q(i)$. The probability $q(i)$ is increased with each unsuccessful attempt to gain access to the channel. After $q(i)$ crosses a threshold, the link drops out and that particular channel is marked as forbidden. We draw inspiration from the binary exponential backoff of the IEEE 802.11 MAC. Probabilistic dropout approaches for links in the context of power control have been used in prior work [18]. A new link continues this process until either its rate requirement is satisfied or it is not possible to select more subcarriers. Fig. 4 shows the steps in our distributed algorithm, which is executed by the transmitter node of each link.

The most important property of our algorithm is that it always results in a feasible subcarrier and bit allocation.

4.3 Illustrative Examples

We present some simple, randomly generated, two link, two channel examples only for the purpose of illustration. Our detailed numerical results with realistic settings are presented in Section 5. The two links have rate requirements $R_1 = 8$ and $R_2 = 14$. The channel gain matrices are as follows:

$$G^1 = \begin{bmatrix} 0.283 & 0.000318 \\ 0.00171 & 0.8829 \end{bmatrix} \times 10^{-4}$$

and

$$G^2 = \begin{bmatrix} 0.2127 & 0.0000581 \\ 0.00019 & 0.1686 \end{bmatrix} \times 10^{-3}.$$

Distributed Algorithm

1. Initializations:

1.a. *q(i) = 0.5*

1.b. Current channel *C = NULL*

1.c. Mark all channels as allowed, *i.e.*, not forbidden

2. while rate not met and at least one channel available

  2.a. Select channel *C* = channel with highest G_i^c / I_i^c factor from channels not marked forbidden

  2.b. Add one bit to channel *C*

  2.c. Perform power control updates for *W* slots

  2.d. If converged

    2.d.1. If rate not yet satisfied go to **2.a.**

    2.d.2. Reset *q(i) = 0.5*

  2.e. Else

    2.e.1. If $q(i) > q_{thresh}$

      2.e.1.a. Drop out of channel *C*; *C = NULL*

      2.e.1.b. Reset *q(i) = 0.5*

      2.e.1.c Mark *C* as forbidden

      2.e.1.d. Go to **2.a.**

    2.e.2. Else

      2.e.2.a. Continue for *W* slots with prob. $1 - q(i)$

        2.e.2.a.1. $q(i) = 1 - (1 - q(i))^2$

        2.e.2.a.2. Go to **2.c.**

      2.e.2.b. Drop out of *C* with prob. *q(i)*; *C = NULL*

        2.e.2.b.1. Reset *q(i) = 0.5*

        2.e.2.b.2. Mark *C* as forbidden

        2.e.2.b.3. Go to **2.a.**

3. Stop

Fig. 4. Distributed Algorithm.

We have the following bit allocations in the two channels:

Bits	MIPA		MSAA		Graph		Distributed	
	Ch 1	Ch 2	Ch 1	Ch 2	Ch 1	Ch 2	Ch 1	Ch 2
Link 1	3	5	0	8	4	4	0	8
Link 2	7	7	8	6	7	7	8	6

The total power (in Watts) is as follows (value of the objective function):

	MIPA	MSAA	Graph	Distributed
Total power	3.278×10^{-6}	5.678×10^{-6}	4.724×10^{-6}	5.678×10^{-6}

For this example, we see that the distributed algorithm achieves an allocation similar to that of MSAA due to its interference avoidance strategy. The graph-based approach tries to utilize the channels equally but does not take into account channel diversity. Clearly, this was an example in which the problem was feasible, i.e., the data rates were achievable. More importantly, the algorithms were able to find feasible allocations.

However, it may be possible that the problem is feasible but the algorithms are unable to find a feasible point. We show another randomly generated example with two links and two channels to show a situation where the problem is feasible, but the MIPA algorithm is unable to find a solution. The rate requirements of the links are $R_1 = 6$ and $R_2 = 8$. The channel gain matrices are as follows:

$$G^1 = \begin{bmatrix} 0.4322 & 0.2901 \\ 0.0158 & 0.0874 \end{bmatrix} \times 10^{-4}$$

and

$$G^2 = \begin{bmatrix} 0.1007 & 0.0118 \\ 0.0000797 & 0.00187 \end{bmatrix} \times 10^{-3}.$$

We have the following bit allocations in the two channels:

Bits	MIPA		MSAA	
	Ch 1	Ch 2	Ch 1	Ch 2
Link 1	2	4	0	6
Link 2	0	0	8	0

In this example, MIPA assigns bits of link 1 to channels 1 and 2 before anything is assigned for link 2. However, once link 1 has been assigned in channels 1 and 2, it becomes infeasible to assign any bits of link 2 to any channel. The algorithm then continues to assign more bits of links 1 and terminates as it is unable to assign any bits of link 2. This is also applicable to the algorithms in [11] and [12].

Before we describe our main results, we illustrate the performance of our algorithms vis-á-vis the optimal solution for small instances of the problem—the two link, two channel case. To find the optimal solution, we enumerate all possible feasible bit allocations and choose the allocation that satisfies the rate requirements and has minimum total power, which is the objective function in problem (9). For the simulated instances, the bit rate requirements of the two links are normal random variables with a mean of eight and a variance of two. Since bit rates have to be integers, the generated random variables are rounded to the nearest integer. For comparison with the optimal solution, we use the following metric:

$$\text{Percentage difference} = \frac{\sum_{i=1}^{2}\sum_{c=1}^{2} P_i^c - \sum_{i=1}^{2}\sum_{c=1}^{2} P_i^c(opt)}{\sum_{i=1}^{2}\sum_{c=1}^{2} P_i^c(opt)} \times 100,$$

where P_i^c is the power of link i and channel c for the algorithm under consideration and $P_i^c(opt)$ is the corresponding power for the optimal solution.

Fig. 5 shows the frequency distribution of the percentage difference for the MIPA, MSAA, and the distributed algorithm, taken over 10,000 different feasible instances of the problem. We see that MIPA finds the optimal solution 75 percent of the time and, at other times, the solution is close to optimal. However, there are some occasions when the problem is feasible but the algorithm fails to find a feasible solution, as was illustrated using an example shown above. The point on the x-axis of Fig. 5 marked as "Failure" shows this event. We also observe that the performance of the distributed algorithm is comparable to that of MSAA despite using only limited local information.

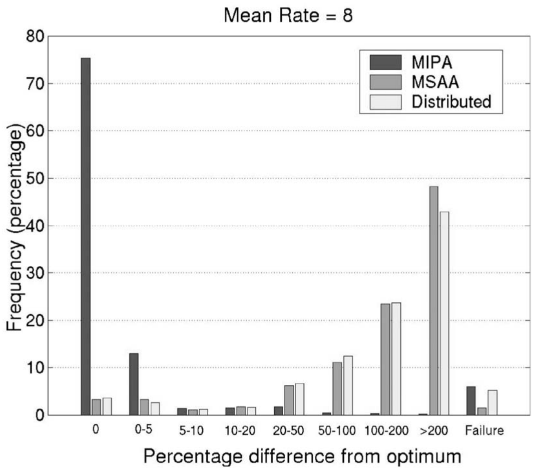

Fig. 5. Comparison with the optimal solution.

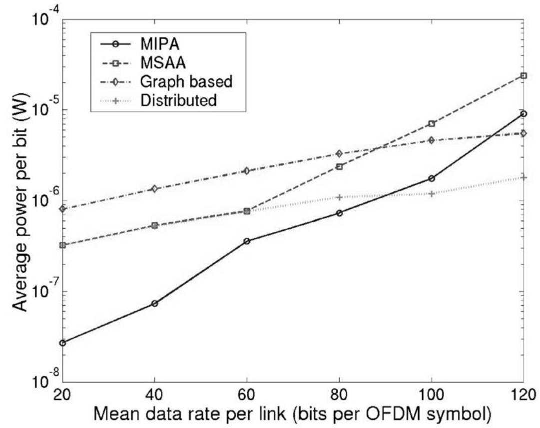

Fig. 6. Average power per bit versus network load.

5 Performance Evaluation and Numerical Results

In this section, we present our simulation results and compare the performance of our MIPA algorithm with that of the MSAA, graph-based algorithm, and the distributed algorithm. In our simulations, we have OFDMA-based wireless networks with 10 links (transmitter-receiver pairs) in an area of 200 m by 200 m. Transmitter locations are generated randomly in the given area with a uniform distribution. Receivers are placed within a 20 m radius of the corresponding transmitter with a distribution that is uniform in both angle and radius. We simulate multipath channels with exponential power-delay profiles, a delay spread of 100 ns [16], and Rayleigh fading. There are 48 subcarriers in the OFDM system with subcarrier spacing equal to 312.5 kHz and the noise power at each subcarrier is 10^{-13} W. The OFDM symbol period is 4 μs, which includes a guard interval duration of 0.8 μs and is kept constant. The path loss exponent is taken to be 4. The bit rate requirements of the links are normal random variables with the mean rate and variance being simulation parameters, which are used to vary the network load. The mean rate is varied from 20 to 120 bits per OFDM symbol and the variance is 20 percent of the mean. A maximum modulation level of 8 bits per symbol was used and the BER is 10^{-6}. The results are averaged over 100 random network topologies. For the distributed algorithm, we choose $W = 10$ slots and $q_{thresh} = 0.95$.

The primary performance metric is the average transmission power per bit over every link. This can be regarded as the price paid for data transmission and is an indicator of the energy efficiency of the rate allocation algorithm. For example, assigning the same set of subcarriers to every link is clearly inefficient and the power transmitted by each link would be very high in order to maintain the SINRs. A more efficient algorithm would try to identify different cochannel sets of links such that the total transmitted power in the presence of interference is reduced. Fig. 6 shows the average power per bit as the network load is varied. The load is increased by increasing the mean rate requirement of each link. As the load increases, more power is needed. This leads to greater interference and, after power control, the transmission power levels needed to maintain the same SINR levels are higher. Hence, the power per bit increases. Since the symbol rate is constant, it means that the energy per bit is higher. However, beyond a certain load, the network cannot support the data rate requirements. This is because, within the same area, it is no longer feasible to load more bits on the limited subcarriers. As a result of this, the total number of admitted bits reach a saturation value. The total transmission power over all links, which is the objective function of our optimization problem, is the product of the average power per bit and the number of bits admitted. At low and medium values of the network load, MIPA shows the best performance. However, at higher loads, the graph-based and distributed schemes appear to perform better. This is because those schemes are not able to fulfill the rate requirements of the links and fewer bits are loaded. As a result, the average power per bit is less. It can be observed that the distributed scheme closely follows MSAA at low loads due to its interference avoidance strategy. However, at high loads, it is unable to successfully satisfy the data rate requirements, even though the rates may be feasible. This is due to the fact that individual links act independently based solely on local knowledge. The number of bits admitted is therefore less and the average power per bit is also less compare to MSAA.

This leads us to study another performance metric—normalized throughput. Normalized throughput is defined as the ratio of the total number of bits admitted to the total number of bits offered. Fig. 7 shows the variation of the normalized throughput with the offered load. As the offered load is increased, the normalized throughput reduces since it is no longer feasible to load more bits of any link on any subcarrier or the algorithms are unable to find a feasible solution. The normalized throughput of MSAA is the best but the power per bit is considerably higher than MIPA. In other words, MSAA can support higher data rates but only by paying a higher price. We see that all our algorithms outperform the graph-based scheme. While the graph-based scheme tries to use low modulation levels, it does not take channel diversity into account and attempts to share all channels among all links. This leads to performance degradation. However, the approach allows flexibility in deciding how to split a link's total data rate

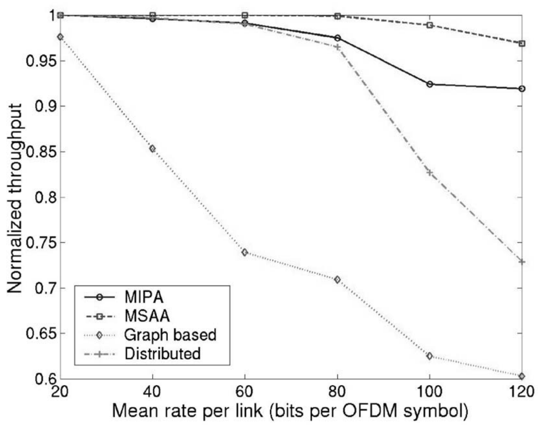

Fig. 7. Normalized throughput versus network load.

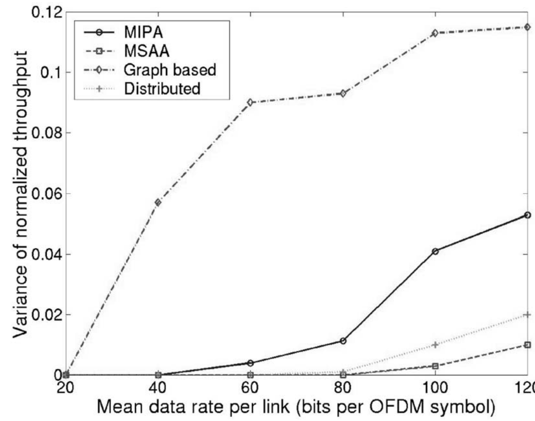

Fig. 8. Variance of normalized throughput.

requirement among different channels. The benefit is that a joint rate allocation, power control, and scheduling problem is reduced to a joint power control and scheduling problem. PCSA [8] is a state-of-the-art technique for the latter.

Fig. 7 shows how the normalized throughput of the entire network varies with the offered load. It does not give any idea about the normalized throughput of individual links. It may happen that certain links may have very high values of normalized throughput while others may be starved in order to have lower average transmitted power per bit. We observed this in a very simple example in the previous section. Fig. 8 shows the variance of the normalized throughput for different values of the offered data rates. This is an indication of the fairness of the algorithms—the lower the variance, the greater the fairness. We see that the MSAA is fairer than the MIPA, but the price paid for that is greater transmission power per bit.

6 Practical Issues

We briefly discuss some practical issues in implementation of these algorithms. Frequency synchronization is essential for the subcarriers in an OFDM system to remain orthogonal. The 5-UP protocol [28] developed by Atheros uses OFDMA in a point-to-multipoint and multipoint-to-point scenario. When the access point transmits, different users extract the frequency offset of the access point relative to their own crystals and lock on. It is shown in [28] that very high frequency accuracy (up to $\pm$ 1 part per million) can be achieved. Another issue is that of timing control. In order for the FFT-based OFDM receiver to function correctly, all signals must arrive within the guard interval. For a guard interval of 800 ns and typical indoor environments, $\pm$ 100 ns timing mismatch is tolerable. We realize that these issues are far more complicated when there are multiple transmitters and receivers. Frequency and timing synchronization are issues under investigation and beyond the scope of this paper. However, we believe that research on resource allocation issues can be carried out in parallel.

Our results are applicable for uncoded modulation. For a highly frequency-selective channel, the performance of uncoded OFDM is unsatisfactory and coded OFDM (COFDM) [29] is used. When coding is performed across multiple subcarriers, the SINR thresholds cannot be independently set for each subcarrier. In COFDM, the receiver consists of a Viterbi decoder. Even for a single point-to-point link, minimizing the BER subject to rate and power constraints has a very high complexity [30]. To mitigate this, the authors of [30] present a two-step suboptimal algorithm for bit and power allocation in which the bit and power loading steps are decoupled. To the best of our knowledge, bit and power allocation for COFDM in the presence of interference is an open problem.

7 Conclusion

In this paper, we have considered the problem of subcarrier and bit allocation for point-to-point links of fixed wireless networks without base stations. The key issues that were considered were location-dependent fading characteristics and the presence of cochannel interference. The objective was to minimize the total transmitted power over all links while trying to satisfy the data rate requirement of each link. This is a combinatorial and constrained optimization problem and we presented centralized heuristic algorithms for allocating rates to the links and at the same time trying to minimize the total transmitted power. In the presence of interference, we argued that the primary performance metric is the average power required for transmitting one bit. This can be regarded as the cost paid for transmitting one bit and is a measure of the efficiency of the rate allocation algorithm.

Later, we presented a scheme for assigning subcarriers and bits to point-to-point links of ad hoc networks in a distributed manner. The key issues that need to be taken into account are the fading characteristics, the interference received by a link, and the interference that a link causes to other links that use the same channels. Power control is needed to maintain the required SINR thresholds for each selected subcarrier. Since a link does not have global knowledge of the network conditions, it needs to wait for a certain time interval to check whether the allocation it made is globally feasible. Thus, there is an inherent latency in rate allocation. We evaluated the performance of our algorithm through extensive simulations and presented several trade offs. The key feature of our distributed algorithm is that it is a simple and practical scheme.

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