Optimal Power Allocation for Cognitive Radio under Primary User’s Outage Loss Constraint

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Abstract—In this paper, we consider a secondary link sharing the spectrum with a primary link in a fading cognitive radio (CR) network. Instead of applying the conventional interference power constraint at the primary user (PU) receiver for the secondary user (SU) to protect the primary transmission, we propose a new constraint on the maximum tolerable outage probability for the PU due to the SU transmission. Under the assumption that perfect instantaneous channel state information (CSI) on the SU channel, the channel from the SU transmitter to PU receiver, and the PU channel is available at the SU transmitter, we derive the optimal power allocation strategies to achieve the ergodic capacity of the SU fading channel. It is shown by simulations that the proposed power allocation strategies can achieve substantial capacity gain for the SU over that based on the conventional interference power constraint, for the same PU outage probability loss.  

I. INTRODUCTION

Cognitive radio (CR) [1] is fast becoming one of the most promising transmission technologies for efficient radio spectrum utilization. Basically, there are two categories of CR operations: opportunistic spectrum access (OSA) and spectrum sharing (SS). OSA is a sensing-based technology, which allows a secondary user (SU) in the CR network to opportunistically access the frequency band originally allocated to a primary user (PU) when the PU transmission is detected to be inactive, while SS allows the SU to transmit simultaneously with the PU over the same frequency even when the PU transmission is active as long as the quality of service (QoS) of the PU transmission is not degraded to an unacceptable level by the interference from the SU.

For SS-based CR networks, a common method to protect the PU is to impose a peak/average interference power constraint to the SU transmission, which requires the interference power received at the PU terminal due to the SU transmission to be below a prescribed threshold, also known as the interference temperature [2]. Motivated by this idea, a great deal of scholarly work has recently appeared in the literature on the design of optimal transmission strategies for the CR. In [3], the authors studied the capacities of different types of single-user and multiuser additive white Gaussian noise (AWGN) SU channels subject to a received-power constraint. For fading PU and SU channels, with perfect channel state information (CSI) on the channels from the SU transmitter to the PU and PU receivers, the optimal power allocation strategy to achieve the ergodic capacity of the SU fading channel was derived in [4] subject to an interference-power constraint at the PU receiver. Then under the same system setup, subject to both the transmit power constraint at the SU transmitter and the interference power constraint at the PU receiver, the optimal power allocation strategies to achieve the ergodic, delay-limited, and the outage capacity of the SU fading channel were studied in [5]. On the other hand, if the additional CSI on the PU fading channel is also known at the SU transmitter, power control for the CR was studied in [6] under the constraint on the maximum ergodic capacity loss of the PU due to the SU transmission. It was shown that the interference-power constraint only achieves an upper bound on the maximum ergodic capacity loss of the PU. The use of the PU ergodic-capacity-loss constraint to protect the PU is more advantageous over the conventional interference-power constraint in terms of maximizing both PU’s and SU’s ergodic capacities. In contrast to using the ergodic capacity as the measure of the performance limit for the PU, in [7] delay-sensitive transmission was considered for the PU and, accordingly, the outage capacity was proposed as the measure of the performance limit. Power allocation strategies for the CR were then studied in [8] under the constraint on the minimum resultant PU outage capacity, where several suboptimal schemes were proposed via the approach of converting the minimum-PU-outage-capacity constraint into an approximate interference-power constraint. Recently, the authors in [9] considered a special case of the minimum-PU-outage-capacity constraint where the PU outage capacity loss due to the SU transmission is limited to be zero.

In this paper, we consider the delay-sensitive transmission for the PU fading channel and thus adopt the outage capacity as the performance measure for the PU. We propose a new type of constraint on the SU transmission to protect the PU transmission by ensuring that the additional outage probability loss (other than that due to the PU’s own fading channel) of the PU due to the SU transmission is no larger than a predefined threshold. We then derive the optimal power control schemes for the SU to maximize the ergodic capacity of the SU fading channel under the proposed constraint together with the SU’s own peak or average transmit power constraint. Numerical examples are also provided to compare the SU’s ergodic capacities under the proposed constraint versus the conventional interference power constraint. Note that our problem formulation includes that in [8] or [9] as a special case.

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The rest of this paper is organized as follows. The system model and the constraint on the PU outage probability loss are introduced in Section II. The optimal power allocation strategies to achieve the ergodic capacity of the SU under the PU outage loss constraint together with the SU peak/average transmit power constraint are presented in Section III. Section IV shows the numerical results on the achievable ergodic capacities of the SU under various power control policies. Finally, Section V concludes the paper.

II. SYSTEM MODEL

In this paper, we consider a simple CR network with one primary link and one secondary link. The primary link consists of a PU transmitter (PU-TX) and a PU receiver (PU-RX), while the secondary link consists of a SU transmitter (SU-TX) and a SU receiver (SU-RX). We assume that the PU and SU links share the same narrow-band frequency for transmission. All the channels involved in the CR network are assumed to be independent block fading (BF) channels. The additive noises at PU-RX and SU-RX are assumed to be independent circularly symmetric complex gaussian random variables denoted by $N_0$. Each of $n_0$ and $n_1$ is assumed to have zero mean and variance of $N_0$, denoted by $CN(0, N_0)$. As shown in Fig. 1, the instantaneous channel power gains at fading state $\nu$ for the primary link, the secondary link, the link from PU-TX to SU-RX, and the link from SU-TX to PU-RX are denoted by $g_{pp}$, $g_{ss}$, $g_{ps}$, and $g_{sp}$, respectively. For brevity, we have dropped the index of the fading block for each of the channel power gains. All the channel power gains involved are assumed to be independent random variables each having a continuous probability density function (PDF). It is also assumed that all the instantaneous channel power gains are available at SU-TX and SU-RX for each fading block.

In this paper, we assume that the PU transmits with a constant power denoted by $P_p$. Therefore, for a given target transmission rate $r_0$, without the presence of the SU, the transmission outage probability of the PU is given by

$$
\varepsilon_p = Pr\left\{ \log_2 \left( 1 + \frac{g_{pp}P_p}{N_0} \right) < r_0 \right\} \quad (1)
$$

where $Pr\cdot$ denotes the probability.

However, when the SU is active and transmits with powers $\{p_s\}$ for different fading blocks, the transmission outage probability of the PU becomes

$$
\varepsilon_c = Pr\left\{ \log_2 \left( 1 + \frac{g_{pp}P_p}{g_{ps}p_s + N_0} \right) < r_0 \right\} \quad (2)
$$

To protect the PU, we require that the additional outage probability of the PU caused by the SU transmission to be no larger than $\Delta \varepsilon$. Mathematically, this constraint can be written as

$$
\varepsilon_c - \varepsilon_p \leq \Delta \varepsilon. \quad (3)
$$

We thus name (3) as the **PU outage loss constraint**.

III. OPTIMAL SU POWER CONTROL UNDER PU OUTAGE LOSS CONSTRAINT

In this section, we study the optimal power control policies for the SU to maximize the SU ergodic capacity under the PU outage loss constraint, together with the average or peak transmit power constraint of the SU, respectively.

A. Average Transmit Power Constraint

First, we consider the average transmit power constraint for the SU. Therefore, the ergodic capacity of the SU can be obtained by solving the following problem (P1):

$$
\max_{p_s \geq 0} \mathbb{E} \left\{ \log_2 \left( 1 + \frac{g_{ss}p_s}{g_{ps}p_s + N_0} \right) \right\} \quad (4)
$$

s.t. $\mathbb{E}\{p_s\} \leq P_{av}$ \quad (5)

$$
\varepsilon_c - \varepsilon_p \leq \Delta \varepsilon \quad (6)
$$

where $\mathbb{E}\{\cdot\}$ denotes the expectation, and $P_{av}$ is the prescribed threshold of the average transmit power of the SU.

From (1), it is observed that when $r_0$, $P_p$ and the distribution of $g_{pp}$ are given, $\varepsilon_p$ is fixed. Therefore, (6) can be simplified to

$$
\varepsilon_c \leq \varepsilon_p \quad (7)
$$

Now, we introduce the indicator function

$$
\chi_c = \begin{cases} 
1, & \text{if } \log_2 \left( 1 + \frac{g_{pp}P_p}{g_{ps}p_s + N_0} \right) < r_0 \\
0, & \text{otherwise.} 
\end{cases} \quad (8)
$$

Then, (7) can be rewritten as

$$
\mathbb{E}\{\chi_c\} - \varepsilon_0 \leq 0. \quad (9)
$$

Now, we apply the Lagrange duality method for P1. Let $\lambda$ and $\mu$ be the nonnegative dual variables associated with (5) and (9), respectively. The Lagrangian of P1 can be written as

$$
L(p_s, \lambda, \mu) = \mathbb{E} \left\{ \log_2 \left( 1 + \frac{g_{ss}p_s}{g_{ps}p_s + N_0} \right) \right\} - \lambda \mathbb{E}\{p_s\} - P_{av} - \mu (\mathbb{E}\{\chi_c\} - \varepsilon_0). \quad (10)
$$

Then, the Lagrange dual function of P1 is expressed as

$$
g(\lambda, \mu) = \max_{p_s \geq 0} L(p_s, \lambda, \mu). \quad (11)
$$

By using the Lagrange dual decomposition method [10], the maximization problem given by (11) can be decoupled...
into parallel subproblems all having the same structure and each for one fading state. For a particular fading state, each subproblem can be shown to be

\[
\max_{p_s \geq 0} \log_2 \left( 1 + \frac{g_{ss}p_s}{g_{ps}P_p + N_0} \right) - \lambda p_s - \mu \chi_c
\]

(12)

where \( p_s \) here denotes the power allocation for this particular fading state.

In order to solve the above problem, we first analyze its objective function. Define

\[
f(p_s) \triangleq \log_2 \left( 1 + \frac{g_{ss}p_s}{g_{ps}P_p + N_0} \right) - \lambda p_s.
\]

(13)

It can be shown that (13) is a concave function with respect to \( p_s \), and \( f(p_s) \) attains its maximum value at

\[
z = \frac{1}{\lambda} - \frac{g_{ps}P_p + N_0}{g_{ss}}\]

(14)

where \((\cdot)^+ = \max\{\cdot, 0\}\).

The indicator function \( \chi_c \) given in (8) is a step function of \( p_s \). Let \( x \) be the critical point of the step function \( \chi_c(p_s) \). Then, \( x \) can be obtained by solving the equation:

\[
\log_2 \left( 1 + \frac{g_{ss}p_s}{g_{ps}P_p + N_0} \right) = r_0.\]

Thus, \( x \) can be expressed as

\[
x = \frac{1}{g_{sp}} \left( \frac{g_{ps}P_p}{2^{r_0} - 1} - N_0 \right).\]

(15)

Let \( p_s^* \) be the optimal solution of each subproblem. The following discussions are made on \( p_s^* \):

**Case 1:** \( r_0 \leq \log_2 \left( 1 + \frac{g_{ss}p_s}{g_{ps}P_p + N_0} \right) \).

In this case, the objective function \( f(p_s) - \mu \chi_c \) has three possible forms which are shown in Fig. 2 (a), (b), and (c), respectively.

If \( x > z \), it is clear from Fig. 2 (a) that \( f(p_s) - \mu \chi_c \) attains its maximum value at \( z \). Therefore, \( p_s^* = z \).

If \( x \leq z \), there are two possible scenarios, which are shown in Fig. 2 (b) and (c), respectively. Therefore, whether \( x \) or \( z \) maximizes the objective function depends on the values of the objective function at these two points. We thus have

\[
p_s^* = z, \quad \text{if } f(x) < f(z) - \mu
\]

\[
p_s^* = x, \quad \text{if } f(x) \geq f(z) - \mu.
\]

**Case 2:** \( r_0 > \log_2 \left( 1 + \frac{g_{ss}p_s}{g_{ps}P_p + N_0} \right) \).

In this case, the relationship between \( f(p_s) \) and \( f(p_s) - \mu \chi_c \) is shown in Fig. 2 (d). Since \( \chi_c \) is equal to 1 regardless of \( p_s \), it is observed that \( f(p_s) - \mu \chi_c \) only has a constant (equal to \( \mu \)) gap from \( f(p_s) \), indicating that \( \chi_c \) has no effect on the optimal solution that maximizes the objective function. Therefore, in this case, \( p_s^* = z \).

Now, we define the following three regions:

\[
R_1 \triangleq \left\{ r_0 \leq \log_2 \left( 1 + \frac{g_{ss}P_p}{N_0} \right), \ x > z \right\}.
\]

\[
R_2 \triangleq \left\{ r_0 \leq \log_2 \left( 1 + \frac{g_{ss}P_p}{N_0} \right), \ x \leq z \right\}.
\]

\[
R_3 \triangleq \left\{ r_0 > \log_2 \left( 1 + \frac{g_{ss}P_p}{N_0} \right) \right\}.
\]

To summarize the results obtained in Cases 1 and 2, the following theorem is obtained:

**Theorem 1:** The optimal solution of P1 is given by

\[
p_s^* = \begin{cases} 
  z, & R_1, \\
  x, & R_2, \ f(x) < f(z) - \mu \\
  z, & R_3 \\
  x, & R_2, \ f(x) \geq f(z) - \mu
\end{cases}
\]

(16)

where \( f(\cdot) \), \( z \), and \( x \) are given in (13), (14), and (15), respectively.

Next, we provide further elaborations for the optimal power control policy of the SU in (16) as follows:

(1) Region \( R_1 \): In this region, since \( x > z \), \( \chi_c(z) \) is zero. This indicates that there will be no outage of the PU even when the SU transmits with the optimal power \( z \). In practice, this happens when the channel condition of the primary link is sufficiently good, and/or the channel power gain of the interference link from SU-TX to PU-RX is sufficiently small, such that the signal power arriving at the PU-RX is much larger than the interference power from the SU. In this case, the interference power from SU-TX is not yet large enough to render the PU into an outage. Therefore, the SU can allocate the transmit power only based on its own channel condition as if there is no PU present.

(2) Region \( R_2 \): In this region, the channel condition of the PU is not as good as that for \( R_1 \). As a result, whether the PU is in an outage depends on the interference power from the SU. If the SU transmits with the optimal power \( z \), it will cause an outage to the PU; if the SU transmits with the power \( x \), it will not do so. Obviously, with the power \( z \) over \( x \), the SU will have a larger transmit rate. However, the SU cannot transmit with \( z \) for each fading state, since it cannot violate the PU outage loss constraint. Therefore, the SU has to choose between the two power allocations: \( x \) and \( z \). If we regard \( \mu \) as the cost or the penalty the SU has to pay when it causes an
outage to the PU, then when the SU transmits with \( x \), there will be no cost incurred; while when the SU transmits with \( x \), it has to pay a penalty \( \mu \) to the objective function. Therefore, the SU has to balance the capacity gain and the cost to PU outage loss constraint for allocating its transmit power.

(3). Region \( \mathcal{R}_3 \): In this region, the PU is always in an outage regardless of \( p_s \). This happens when the channel condition of the primary link is very poor, and the PU will be in an outage even if the SU does not transmit. In this case, the transmission of the SU will not cause any additional outage loss to the PU transmission. Therefore, the SU can allocate the transmit power based solely on its own channel condition.

**Special Case:** It is seen from (16) that if \( \mu \) goes to infinity, \( f(x) \geq f(z) - \mu \) will always be true. Therefore, for the whole region of \( \mathcal{R}_2 \), the SU always transmits with power \( x \), and it will not cause any additional outage probability loss to the PU. In this case, (16) reduces to the optimal power allocation for \( \Delta \varepsilon = 0 \).

### B. Peak Transmit Power Constraint

Next, we consider the peak transmit power constraint for the SU. In this case, the ergodic capacity of the SU can be obtained by solving the following problem (P2):

\[
\max_{p_s \geq 0} \quad \mathbb{E}\left\{ \log_2 \left( 1 + \frac{g_{sp}p_s}{g_{sp}p + N_0} \right) \right\} \tag{17}
\]

s.t. \( p_s \leq P_{pk} \)

\( \varepsilon_c - \varepsilon \leq \Delta \varepsilon \tag{18} \)

where \( P_{pk} \) is the prescribed threshold of the peak transmit power of the SU.

It is observed that P2 is similar to P1. Therefore, similar methods for P1 can be applied to solve P2. Let \( \mu \) be the nonnegative dual variable associated with the constraint given by (19). By adopting the indicator function \( \chi_c \) given in (8), it can be shown that P2 can be decomposed into the following subproblems each for a different fading state:

\[
\max_{0 \leq p_s \leq P_{pk}} \quad \log_2 \left( 1 + \frac{g_{sp}p_s}{g_{sp}p + N_0} \right) - \mu \chi_c. \tag{20}
\]

Note that \( p_s \) here refers to the power allocation for a particular fading state. Define

\[
h(p_s) \triangleq \log_2 \left( 1 + \frac{g_{sp}p_s}{g_{sp}p + N_0} \right). \tag{21}\]

The above subproblem can be solved based on the following discussions:

**Case 1:** \( r_0 \leq \log_2 \left( 1 + \frac{g_{pp}P_p}{g_{pp}P_p + N_0} \right) \).

In this case, the indicator function \( \chi_c \) is always equal to 0 for any \( p_s \in [0, P_{pk}] \). Obviously, (20) is maximized when \( p_s = P_{pk} \) due to the fact that \( h(p_s) \) is an increasing function. Therefore, \( p_s^* = P_{pk} \).

Intuitively, \( r_0 \leq \log_2 \left( 1 + \frac{g_{pp}P_p}{g_{pp}P_p + N_0} \right) \) corresponds to the scenario when the channel condition of the primary link is sufficiently good or the interference link from SU-TX to PU-RX is sufficiently weak. In this case, the PU will not be in an outage even if the SU transmits with the peak power.

**Case 2:** \( \log_2 \left( 1 + \frac{g_{pp}P_p}{g_{pp}P_p + N_0} \right) < r_0 \leq \log_2 \left( 1 + \frac{g_{pp}P_p}{g_{pp}P_p + N_0} \right) \).

Let \( x \) be the solution of \( \log_2 \left( 1 + \frac{g_{pp}P_p}{g_{pp}P_p + N_0} \right) = r_0 \). Thus, \( x \) is given by (15). It is noted that \( 0 \leq x < P_{pk} \) in this case.

In this region, the value of the step function \( \chi_c(p_s) \) depends on \( x \). If \( p_s > x \), \( \chi_c = 1 \); otherwise, \( \chi_c = 0 \). Obviously, we have

\[
p_s^* = x, \quad \text{if} \quad h(x) \geq h(P_{pk}) - \mu
\]

\[
p_s^* = P_{pk}, \quad \text{if} \quad h(x) < h(P_{pk}) - \mu. \tag{22}\]

The intuitive interpretations of the above two subcases are as follows. Here, the SU can choose to cause an outage to the PU or not. If it chooses to do so, it has to pay the cost \( \mu \). Thus, to transmit with \( P_{pk} \) or \( x \) depends on the balance between the SU capacity gain and the cost to the PU outage loss. If the capacity gain \( h(P_{pk}) - h(x) \) is larger than the cost \( \mu \), the SU should choose to transmit with \( P_{pk} \) by causing an outage to the PU; otherwise, it should transmit with a smaller power \( x \) by avoiding the PU outage.

**Case 3:** \( r_0 > \log_2 \left( 1 + \frac{g_{pp}P_p}{g_{pp}P_p + N_0} \right) \).

In this case, the indicator function \( \chi_c \) is always equal to 1. Since \( h(p_s) \) is an increasing function with \( p_s \), it is easy to observe that (20) is maximized by \( p_s^* = P_{pk} \).

Intuitively, for this case, the PU is in an outage even if there is no interference from the SU. Therefore, the transmission of the SU will not cause any additional outage to the PU. Hence, the SU should transmit with its peak power to maximize the transmit rate.

Now, we define the following three regions:

\[
\mathcal{R}_1 \triangleq \left\{ r_0 \leq \log_2 \left( 1 + \frac{g_{pp}P_p}{g_{pp}P_p + N_0} \right) \right\},
\]

\[
\mathcal{R}_2 \triangleq \left\{ \log_2 \left( 1 + \frac{g_{pp}P_p}{g_{pp}P_p + N_0} \right) < r_0 \leq \log_2 \left( 1 + \frac{g_{pp}P_p}{g_{pp}P_p + N_0} \right) \right\},
\]

\[
\mathcal{R}_3 \triangleq \left\{ r_0 > \log_2 \left( 1 + \frac{g_{pp}P_p}{g_{pp}P_p + N_0} \right) \right\}.
\]

With the results obtained in the above three cases, the following theorem can be obtained:

**Theorem 2:** The optimal solution of P2 is given by

\[
p_s^* = \left\{ \begin{array}{ll}
P_{pk}, & \mathcal{R}_1, \\
P_{pk}, & \mathcal{R}_2, \quad h(x) < h(P_{pk}) - \mu \\
P_{pk}, & \mathcal{R}_3, \quad h(x) \geq h(P_{pk}) - \mu \
\end{array} \right.
\]

where \( h(\cdot) \) and \( x \) are given in (21) and (15), respectively.

**Special case:** It is observed that when \( \mu \) goes to infinity, \( h(x) \geq h(P_{pk}) - \mu \) will always be true. In this case, the SU will not cause any additional outage loss to the PU, and (22) will reduce to the power allocation strategy for \( \Delta \varepsilon = 0 \), the same as that obtained in [9].

### IV. Simulation Results

In this section, simulation results are given to evaluate the performance of the proposed power allocation strategies. All the channels involved are assumed to be Rayleigh fading, and...
the channel power gains are thus exponentially distributed with unit mean. For these simulations, the noise power $N_0$ is assumed to be 1. The transmit power $P_p$ of the PU is assumed to be 15 dB, and the target rate $r_0$ of the PU is chosen as 1 bit/complex dimension (dim.). Therefore, without the presence of the PU, the outage probability $\epsilon_p$ of the PU is around 3.1%.

Fig. 3 compares the ergodic capacities of the SU under the interference power constraint with those under the proposed constraint. For fair comparison, the interference power threshold is chosen such that the additional outage of the PU under the interference power constraint is same as that under the proposed constraint. It is observed that in the case of $\Delta \epsilon = 0.1$, the capacity gain is very small when $P_{pk}$ is small. However, with the increase of $P_{pk}$, the capacity gain gradually becomes large. This suggests that the proposed constraint is more effective for large $P_{pk}$. Moreover, when no additional PU outage is allowed, i.e., $\Delta \epsilon = 0$, the SU transmission is not allowed under the interference power constraint. However, under the proposed constraint, the SU transmission is not only allowed, but also achieves a consistent increase of capacity with $P_{pk}$. These observations show the superiority of the proposed constraint over the interference power constraint.

Fig. 4 shows the SU ergodic capacities under peak/average transmit power constraint for different values of $\Delta \epsilon$. The proposed SU power control is applied here. It is observed that when $\Delta \epsilon$ is small, the SU capacity increases with $\Delta \epsilon$. However, when $\Delta \epsilon$ is sufficiently large, the SU capacity gets saturated. It is also observed that $\Delta \epsilon$ needed for the capacity to get saturated under small transmit power constraint is smaller than that under large transmit power constraint. This is due to the fact that when $\Delta \epsilon$ is sufficiently large, the PU outage loss constraint will be inactive, and the SU capacity is only determined by its transmit power constraint. Furthermore, it is observed that for the same $\Delta \epsilon$, the SU capacity under average transmit power constraint is larger than that under peak constraint. This is true since the power allocation strategy under average power constraint is more flexible.

V. CONCLUSION

In this paper, a new method to protect the delay-sensitive primary user transmission is proposed for a spectrum-sharing-based fading cognitive radio network. The newly proposed method protects the PU by ensuring that the additional outage probability drop of the PU due to the SU transmission is no larger than a predefined threshold. With the additional CSI on the PU fading channel, the optimal power allocation strategy to achieve the ergodic capacity of the SU link is derived under this new constraint as well as the peak/average transmit power constraint for the SU. It is shown that for the same resulted PU outage probability loss, substantial capacity gains can be achieved for the secondary link under the proposed method over the conventional interference power constraint.

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